Physics-informed machine learning in prognostics and health management: State of the art and challenges

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Abstract

Prognostics and health management (PHM) plays a constructive role in the equipment's entire life health service. It has long benefited from intensive research into physics modeling and machine learning methods. However, in practice, the existing solutions often encounter difficulties caused by sparse data & incomplete system failure knowledge. Pure machine learning or physics-based methods can sometimes be infeasible in such situations. As a result, there has been a growing interest in developing physics-informed machine learning (PIML) models which allow incorporating different forms of physics knowledge at different positions of the machine learning pipeline. This combination provides significant assistance for detection, diagnostics, and prognostics. However, to the best of our knowledge, the bibliometrics analyses and the comprehensive review of the existing research concerning PIML in PHM remain vacant. Our review is therefore dedicated to filling these gaps. We synthesize the concept of PIML in PHM, and propose a taxonomy of PIML approaches from the perspective of "Expression forms of informed knowledge" and "Knowledge informed methods". The findings and discussions presented in this paper enable us to clarify the current state of the art and the emerging opportunities of PIML approaches, especially for building PHM systems that can work under the "small data and scarce physics knowledge" paradigm.

Keywords: `Physics-informed machine learning, Prognostics and health management, Physics-informed input space, Physics-constraint learning, Physics-embedded algorithm structure, Knowledge.`

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Abbreviations		
ANN	:	Artificial Neural Network
CNN	:	Convolutional Neural Network
CRA	:	Cumulative relative accuracy
DNN	:	Deep Neural Networks
FCN	:	Fully Convolutional Networks
GNN	:	Graph Neural Networks
KSVD	:	K Singular Value Decomposition
LSTM	:	Long short-term memory
MAE	:	Mean Square Error
MSE	:	Mean Absolute Error
NMSE	:	Normalized Mean Absolute Error
NMAE	:	Normalized Mean Square Error
NODE	:	Neural Ordinary Differential Equations
PBM	:	Physics based methods
PCA	:	Principal Component Analysis
PDE	:	Partial differential equation
PHM	:	Prognostic and Health Management
PIML	:	Physics-informed Machine Learning
RMSE	:	Root mean square error
RNN	:	Recurrent Neural Network
ROM	:	Reduced Order Model
RUL	:	Remaining useful life
SVM	:	Support Vector Machine
VAE	:	Variational AutoEncoder
DRM	:	Disk-ram

1. Introductions

Prognostics and Health Management (PHM) is an interdisciplinary engineering discipline. It ensures the real-time health assessment and future state prediction of systems based on up-to-date information and data processing [1] by two main paradigms: data-driven and physics model-based methods (PBM).

Enabled by machine learning (ML), with a recent surge related to deep learning [2, 3], data-driven methods extract features from data and identify the underlying degradation processes, showing promising results at different failure scales from macro to micro degradation and damages [4]. However, ML in PHM faces three dilemmas:

1. First dilemma arises from the limited data quality. One can cite: a) sparse & noisy observations caused by limited structural conditions and pervasive monitoring instrument costs [5]; b) sparse & noisy failure information due to restricted monitoring time and high run-to-failure operation costs [6]; and c) sparse labels caused by limited expert knowledge and high manual labeling cost [7].

2. Second dilemma is the opaque, unexplained nature of ML, leading to the trust deficit [8]. For high investment and risk industrial domains, the unobservable process between the ML input data and the output PHM results is viewed as a "black box" with interpretability difficulties [9].

3. Third dilemma in ML arises from the lack of physics consistency. ML generally converges in the direction that best fits the training samples which might not absolutely conform to the Physics-constraints [10].

In contrast to ML, PBM observes failure phenomena and then establishes mathematical or numerical mechanism models to represent faults or degradations [11]. When the failure natures are well understood, PBM needs fewer data than ML [12] and achieves better generalization [13]. However, modern engineering systems are complex and influenced by mutual non-linear interactions among the subsystems. Thus, the PBM performance can be affected by different factors such as the system scale [14] and complexity [15], leading to the following dilemmas:

1. First dilemma concerns the epistemic uncertainties in the model simplification and the paradox between the computational efficiency and the PBM's credibility [16].

2. Second dilemma is the sometimes limited understanding of the fault mechanism to construct trustworthy degradation models [17].

3. Third dilemma arises from the unknown and unobservable parameters of the PBM.

Due to the above dilemmas, the purely physics-based models are rarely applied in practice [18, 19].

Regarding limitations of both data-driven and physics-based methods, it is necessary to develop hybrid approaches to overcome the drawbacks and inherit the advantage of each one. Fig. 1 shows the motivation for a combination of PBM and ML models. In fact, physics-informed machine learning (PIML) is a promising solution in the case of sparse data and incomplete physics knowledge. PIML is formally introduced at the Los Alamos PIML workshops during 2016-2020 [20] with the initialization in solving complex physics problems by ML. Particularly, Raissi *et al.* [21] proposed a physics-informed neural network (PINN)-based Partial differential equations solution, leading to a boom in "informed NN". Meanwhile, many industrial partners, including GE[22], IBM [23], Nvidia [24], US DARPA [25] and NASA [26], the Argonne National Laboratory [27] and Siemens [28], have paid high attention to the application of PIML. 2blueThe motivation behind the development of PIML has been extensively discussed in existing literature, highlighting its inherent strengths and advantages as follows:

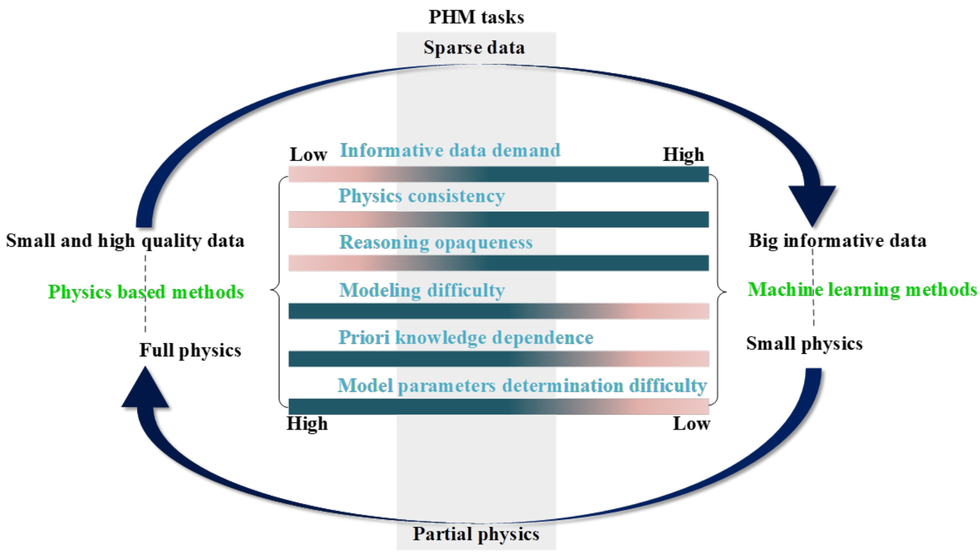


Figure 1: Sparse data and incomplete physics knowledge drive PHM techniques towards the combination of PBMs and ML.

1. PIML merges data-driven machine learning and physics principles to boost the precision and interpretability of prognostics and health management (PHM) system predictions.

2. PIML shines in tackling intricate dynamics of complex and nonlinear systems in PHM applications. It achieves this by integrating physics-based constraints and equations, thereby enhancing its predictive and modeling abilities.

3. The blend of physics-based knowledge enables PIML to generalize more effectively, offering reliable predictions even beyond observed data. This is particularly useful in scenarios with sparse or incomplete training data.

4. A key advantage of PIML models is their improved interpretability owing to the explicit incorporation of physics. By embedding physical constraints and equations, PIML allows to more easily elucidate some of the underlying mechanisms that drive system behavior, which aids diagnostic analysis and decision-making.

5. Despite limited data, PIML's efficient use of system physics knowledge allows for accurate predictions, reducing the dependence on large datasets and potentially minimizing data acquisition costs.

6. PIML is robust to noise and outliers due to its enforcement of physical laws, which can filter out erroneous or noisy data, leading to dependable predictions.

7. PIML provides computational benefits by amalgamating physics-based constraints with machine learning algorithms, thereby yielding efficient models that reduce computational complexity, suitable for real-time or near real-time applications.

Inspired by these advances, there have been several studies on PIML for anomaly detection, fault diagnostics, and prognostics [29]. 2blue Compared to other potential solutions for sparse data, like transfer learning [30], the advantage of PIML in PHM is in assisting data-driven insights, utilization of expert knowledge, adaptability, and scalability. By leveraging these strengths, PIML can also enhance the other methods, and the capabilities of addressing the PHM challenges [31] in limited data availability, complex and nonlinear system dynamics, physics consistency related to trust, handling of noise and uncertainty, integration of multi-source and heterogeneous data, and transferability across systems and domains [32]. Furthermore, PIML is not in competition with these methods, but is the icing on the cake to achieve win-win situations, for example through approaches such as PI-transfer learning in aerospace anomaly detection [33], and PI-meta learning for machining tool wear prediction [34]. Although many works are exciting, to the best of our knowledge, none of the existing papers provides a comprehensive review of PIML in PHM. In addition, no overall qualitative and bibliometrics analyses are conducted. Finally, taxonomy and applications in PHM are unclear and waiting for discussion in particular with respect to: I) ***Expression forms of Informed Knowledge*** and II) ***Knowledge Informed Methods***. Therefore, this paper aims to fill the above-mentioned gaps. Besides, the open challenges toward the maturity of PIML in PHM are also highlighted.

The rest of the paper is organized as follows. Section. 2 presents a bibliometric analysis of the existing works concerning PIML in PHM and thus shows an overview of the research trend on this topic. Section. 3 provides a comprehensive and insightful review of PIML methods in PHM. Section. 4 aims to summarize and discuss the source of physics knowledge, which can be used to derive ML models, as well as the corresponding integration methods. Finally, Section. 5 summarizes the main contribution of this paper and provides insights into potential future research.

2. Bibliometrics analysis

This section aims to provide an overview of the research interests of PIML studies in the field of PHM. Firstly, we describe the literature research methodology in Subsection. 2.1. Then, based on the bibliometric analysis of PIML in PHM, the research trend on this topic is discussed in Subsection. 2.2. Finally, Subsection. 2.3 compiles a statistical analysis of the works on PIML applied to PHM.

2.1. Literature research methodology

The bibliographic data investigated in this work covers the period from January 2013 to January 2023. 2blueThe time span chosen in this article is based on the understanding that PIML technology emerged around 2016. However, upon investigating the research trend from hybrid frameworks to PIML, we found similar technical concepts dating back to 2013. Consequently, we conducted a literature search spanning from 2013 to the present. In the search flowchart presented in Fig. 2, the survey is simultaneously retrieved from Web of Science (WoS) and Google Scholar. The search on Google Scholar is to verify the adequacy of the search in WoS.

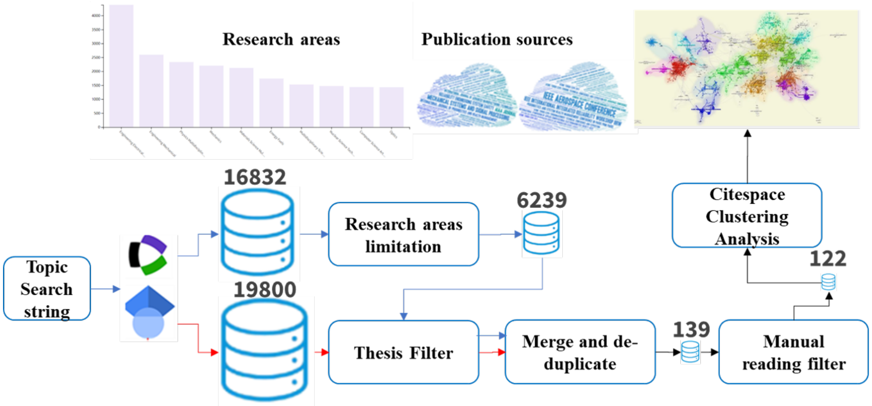


Figure 2: Search methodology flowchart.

"Topic search strings" are defined as all terms in *(Topic 1)* AND *(Topic 2)*. where ***TOPIC 1:*** *"Physics-informed" OR "Physics guided" OR "Physics induced" OR "Physics aware" OR "Physics infused" OR "Domain knowledge" OR "Hybrid framework" OR "Hybrid method".*

AND **TOPIC 2:***"Machine learning" OR "Deep learning" OR "Data-driven"*

In a further filtering, the "Topic filter" consists of "*Core vocabulary*", and "*Interfering words:*"

Core Vocabulary:*"Detection" OR "Diagnostic" OR "Prognostics" OR "Failure" OR "Remaining useful life" OR "Prediction" OR "Identification" OR "Classification" OR "SHM" "Damage" OR "Deterioration" OR "Recognition" OR "Fracture" OR "Crack" OR "Deformation" OR "Abnormal" OR "Equipment" OR "Bearing" OR "Gear" OR "Power".*

Interfering words: *Not "Language" OR "Medical" OR "Cancer" OR "Face" OR "Emotion" OR "Text".*

The first search result provides more than 36632 manuscripts from the two largest databases: Google Scholar and Web of Science. Then, we limited the search to the areas where engineering PIML and PHM solutions are usually implemented, such as Electronics, Aerospace, Mechanics, Computer Science, Engineering Multidisciplinary, Automation Control Systems, Energy Fuels, Engineering Civil, Engineering Manufacturing, etc. After this step, 6239 papers are kept. To yield insight into the published material list, we implemented further selection steps like "Thesis Filter", "Manual screening", and "Merge duplication". In "Thesis Filter", we perform topic filtering by the **Interfering words** and **Core Vocabulary** mentioned above, and then the results of the filtering are manually reviewed in "Manual screening" to determine that the article topic fits within the scope of the review. By doing this, we found that only 139 papers have the topic with the PIML-related hybrid framework in PHM. Among them, 122 papers discuss PIML in PHM in detail. These papers are exploited to draw critical remarks on the research trends as well as interesting statistical results on the development of PIML in PHM.

2.2. Research trend analysis of PIML in PHM

2blueTo have an overview of the research trend from the hybrid frameworks to PIMLs, in this section, one can see that during only a decade of development, research related to the combination of model-driven and data-driven methods in the industry has appeared in a wide range of conferences and scientific journals, as shown in Fig. 3. One can see that Mechanical Systems and Signal Processing (MSSP) journal has published a large number of manuscripts on this hybrid framework with more than 573 papers. IEEE Aerospace is the conference attracting the most related hybrid framework studies, with 706 papers. Next, to show an overview of the evolving process from the hybrid

(a) Conferences

(b) Journals

Figure 3: Publication sources on hybrid frameworks considered in this review.

framework to the PIML methods in PHM, we conducted a bibliometric analysis by using cite space software [35]. Particularly, this software allows automatically analyzing the keyword co-occurrence and then generating the clustering network of the most widely used keywords (Fig. 4) and its development trend over time (Fig. 5).

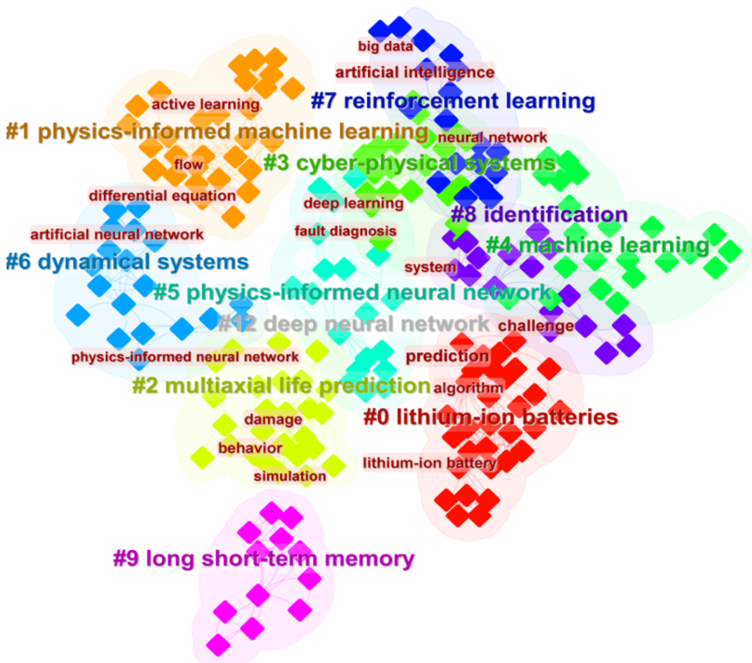


Figure 4: Keywords co-occurrence clustering.

From the clustering network in Fig. 4, it appears that the most widely used keyword is "physics-informed machine learning". Associated with this keyword, one can cite "active learning" and "differential equation" techniques that are used to build the PIML framework. Besides, "physics-informed neural network" is also a critical keyword that has co-occurred with "dynamical systems" and "deep neural network". Looking into the relevant studies, one can see that PINN is usually used to capture system dynamic behaviors for damage detection, fault diagnostics, and failure prediction.

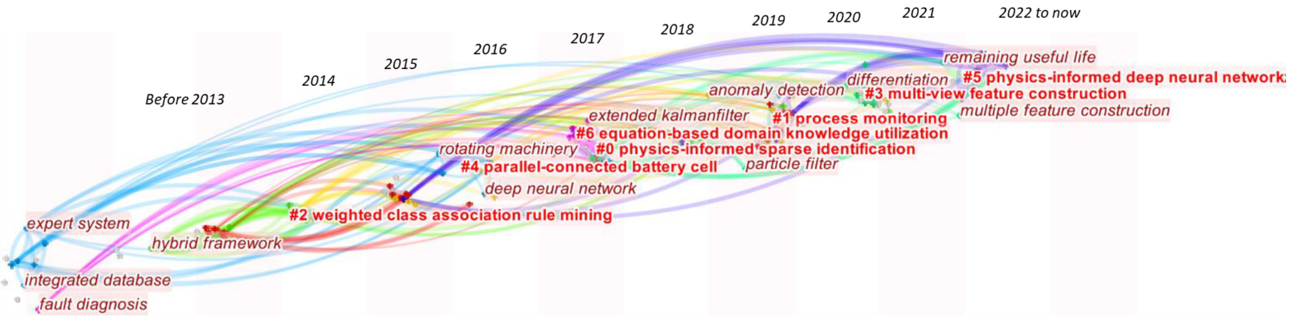


Figure 5: Development trend of the keywords in Knowledge-assisted PHM studies.

Considering Fig. 5, one can see that the development trend of the keywords, which are used in PIML research, shifts from the expert system (in 2011), weighted class association rule mining (in 2014), to PINN in recent years. Meanwhile, one can notice an increasing demand for physics knowledge, which is represented by the often-occurred keywords such as "physics-informed sparse identification" and "equation-based domain knowledge utilization". Next, "deep neural networks" and "extended Kalman filter" (or "particle filter") are usually combined to create PIML framework ([36]). Besides, research related to "embedding differential equations" of lifetime degradation in ML is also highlighted through this trend analysis ([37]).

2.3. *Statistical analysis of PIML in PHM*

2blueThis section aims to discuss the results of the statistical analysis of existing papers relating to PIML in PHM.

2.3.1. *Existing terminologies*

There are numerous terminologies similar to "physics-informed machine learning" (see Table. 1). According to the statistical results of all publications relating to PIML in PHM, the distribution of those terms are: "*Physics-informed*" (47.1%), "*Physics based*" (19.9%), "*Physics guided*" (18.3%), "*Physics infused*" (8.8%), and "*Physics aware* (5.9%)".

Table 1: Summary of the existing terminologies relating PIML in PHM

Terminology	References	Total number
Physics infused	[27], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48]	12

Physics based	[14], [49], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], [62], [63], [64], [65], [10], [32], [66], [67], [68], [69], [70], [71]	26
Physics guided	[72], [73], [74], [75], [76], [77], [78], [79], [80], [81], [82], [83], [84], [85], [86], [87], [88], [89], [90], [91], [92], [93], [94]	23
Physics aware	[95], [96], [97], [98], [99], [100], [101], [102]	8
Physics-informed	[103], [104], [105], [106], [107], [108], [109], [110], [20], [111], [112], [113], [114], [115], [116], [117], [118], [119], [120], [121], [122], [123], [124], [125], [126], [127], [128], [129], [130], [81], [123], [131], [132], [133], [134], [135], [136], [137], [138], [37], [21], [139], [140], [141], [142], [143], [144], [145], [146], [147], [117], [148], [149], [150], [151], [152], [153], [154], [155], [156], [157], [158], [109], [159]	64

The heterogeneity of those terms can pose a major obstacle to research on this topic as well as its wide application in practice. Therefore, in this Subsection, we seek to clarify the similarities and differences between the existing terminologies.

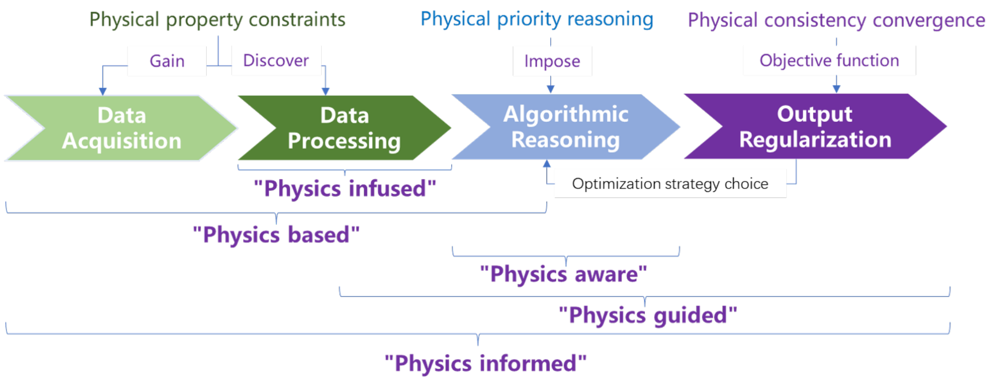


Figure 6: Different terminologies' focus.

2blue Fig. 6 presents the main scope of each terminology, and it can be summarized as follows:

1. "Physics infused" aims to discover and incorporate physics property constraints in the data preprocessing [160, 54], leveraging physics-derived parameters and relations to enhance the performance of ML models, especially in sparse data scenarios [161].

2. "Physics based" focuses on the integration of physics models or constraints in the model-data hybrid framework [68, 162], incorporating physical principles in feature engineering, system modeling, and constraint-based approaches [101].

3. “Physics aware” emphasizes perceiving the intrinsic behavior and structural features of the system [95], aligning the ML algorithm structure or interaction structure with the physical system to achieve consistency in physics.

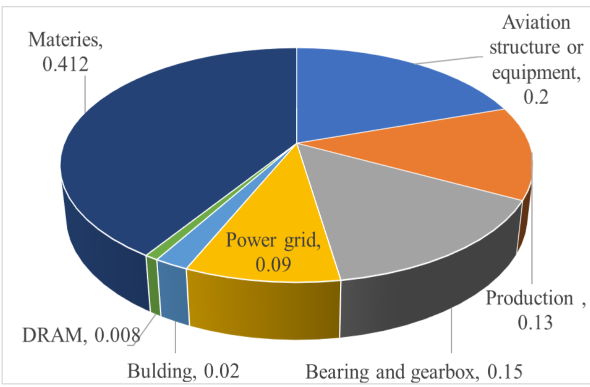
4. “Physics guided” expands the focus of “Physics aware”on visually representing degradation states or using physics knowledge to guide the data processing [79, 163], the design of ML structures, algorithmic weights and biases [92], or empirical loss functions [89].

5. “Physics-informed” refers to the broadest framework [164] that covers the entire machine learning process, incorporating physics knowledge in various aspects along the machine learning pipelines.

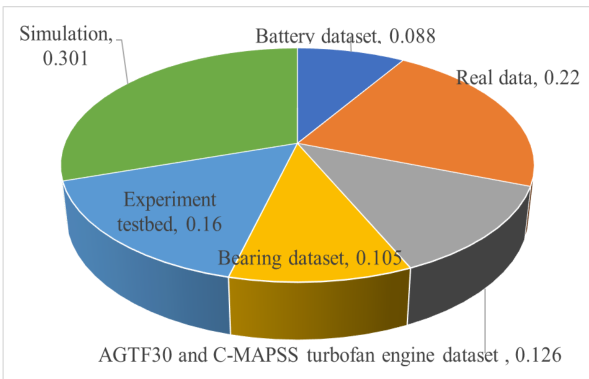
Therefore, in this review, the term “PIML” (Physics-informed Machine Learning) is chosen as the discussed terminology to encompass the integration of physics knowledge within machine learning approaches.

2.3.2. *Application areas, main motivations, and methods’ evaluation metrics*

Fig. 7 presents the distribution of application areas and data sources of the studies on PIML in PHM. From Fig. 7 (a), one can see that most of the current PIML studies in PHM focus on materials damage (41.2%) because there already exists in this area numerous studies in mathematical and physical modeling of material dynamic behaviors. These studies provide a solid foundation for the rapid development of PIML models. Other applications such as aviation structure and equipment (20.0%), production equipment (13.0%), bearing and gearbox (15.0%), and power grid (9%) also attract more attention from the research community in recent years. Besides, considering data sources Fig. 7 (b), we find that most data sources for PIML studies come from simulation (30%). Also, the most used bench-marking datasets are Turbo engine simulation dataset (C-MAPSS and AGTF30) and battery dataset (Oxford and NASA). The studies of PIML models for real systems are limited to small experimental platforms (16%). Those observations can be explained by the lack of exploitable physics-based knowledge of real systems that are usually difficult to model.



(a) Application cases



(b) Data sources

Figure 7: Statistical results of main application areas and data sources of PIML in PHM.

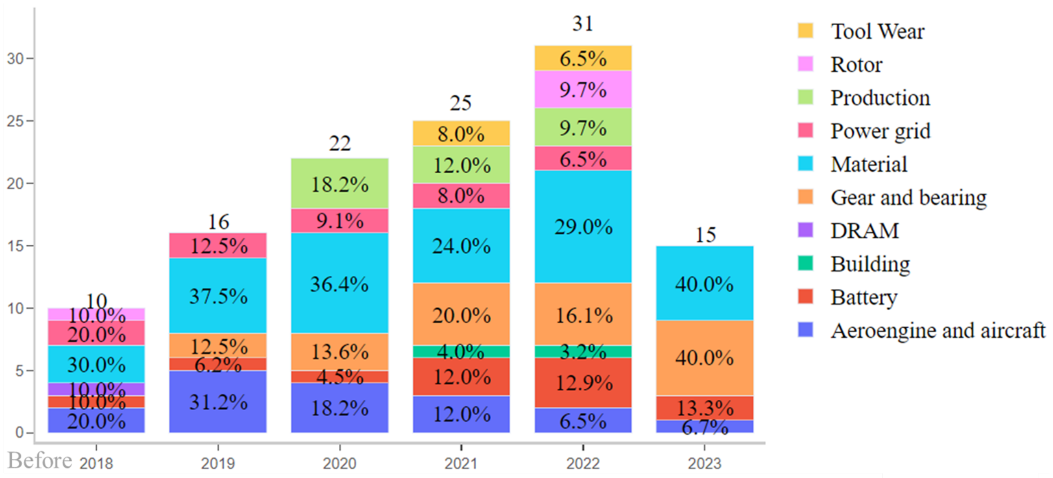


Figure 8: Overview on applications of PIML in PHM.

2blueFig. 8 shows the number of publications concerning PIML in PHM per year. It highlights an increasing interest of the research community in this topic. One can see that the number of publications per year significantly increased after 2020. The research in materials, bearings, gears, aerospace structures, and power systems has garnered significant attention.

Fig. 9 presents statistical results of relevant research with respect to data quality. One can see that a large proportion of PIML research focuses on solving the PHM tasks in the presence of sparse (26%) or noisy data (38%). This remark highlights the relevance of the PIML over purely data-driven models when it comes to such data quality issues.

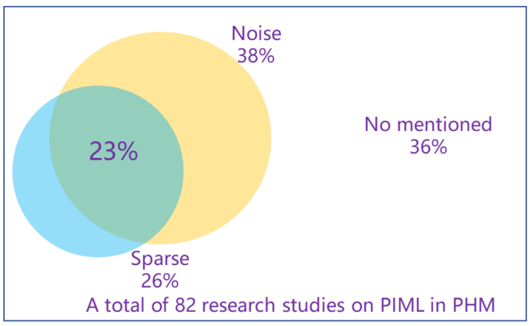


Figure 9: Statistical results of relevant research on data quality problem.

Table. 2 lists the metrics used in the literature to train and evaluate the performance of PIML methods. It also shows the specific PHM tasks to which these metrics correspond, as well as the types of monitoring measurements.

Table 2: Summary of the training, testing metrics and monitoring signals for PIML according to PHM tasks

Ref.	Train metric	Test metric	PHM tasks	Signals
[161]	MSE	MSE	Condition monitoring	Displacement and voltage

[124], [131]	MSE	MSE	Condition monitoring	Vibration
[165]	MSE	MSE	Condition monitoring	Currents, voltages and time measurements
[106]	MAE, MSE, RMSE	Precision, recall, F1-score	Condition monitoring & Fault diagnostic	Vibration, acoustic, image, temperature
[53], [74], [151]	MSE	MSE	Condition monitoring & Fault diagnostic	Stress
[126], [166], [132]	MAE, Cross-entropy loss	Precision, recall, f-k value, accuracy, macro F1, and G-mean	Condition monitoring & Fault diagnostic	Power, voltages, current
[13]	Customized design loss	RMSE	Condition monitoring & Fault diagnostic	Temperature, pressure, vibration, and air flow
[167], [168]	Cross-entropy	Relative percentage error	Condition monitoring & Fault diagnostic	Vibration
[169]	MAE	Confusion matrix	Fault diagnostic	Temperature
[170]	Maximum cross entropy	MAE	Fault diagnostic	Vibration
[171], [172]	Binary cross-entropy	Categorical cross-entropy	Fault diagnostic	Vibration
[173]	MAE, similarity distance	Precision, recall, f-measure, confusion matrix	Fault diagnostic	Temperature, pressure, and fuel coefficient
[14]	Customized design metrics	Test false positive rate, MAE.	Fault diagnostic	Vibration, acoustic signal, and temperature
[6]	MAE	Confusion matrix, recall, precision	Fault diagnostic	Vibration
[50]	MSE	MSE, Pearson correlation coefficients test	Fault diagnostic	Vibration
[112], [146]	Cross-entropy loss	Confusion matrix	Fault diagnostic	Vibration, strain, torque, acoustic emission
[130]	MSE	MAE	Fault diagnostic	Magnetic flux leakage image, stress
[117]	Customized loss	Confusion matrix	Fault diagnostic	Far-field loads, stress ratio and a corrosivity index
[129], [133]	MSE	MAE	Fault diagnostic	Stress
[81]	MSE, Kernel norm	Relative percentage error	Fault diagnostic	Ultrasonic signal
[27]	MSE	MSE	Fault diagnostic	Stress, temperature
[143]	Cross-entropy loss	MAE	Fault diagnostic	Vibration
[150]	Customized loss	Customized metric	Fault diagnostic	Wave data
[174]	Cross-entropy	Confusion matrix	Fault diagnostic	Stress
[175]	Softmax loss test	Relative percentage error,	Fault diagnostic	Guided wave signal
[37]	MSE	MSE, Pearson correlation coefficients test	Fault diagnostic	Acoustic signal
[84]	Cross-entropyMSE, Softmax loss	MAE	Fault diagnostic	Mode shapes signal
[154]	Customized loss	F1 score	Fault diagnostic	Proposed access location, error locations

[176]	MAE	Relative percentage rate	Fault diagnostic	Stress
[70]	Cross-entropy loss function	MAE	Fault diagnostic & RUL prediction	Phase field images
[36]	MAE	$\alpha_\lambda$distribution accuracy	RUL prediction	Vibration
[51]	MAE	One σ tolerance interval	RUL prediction	Voltage and current
[54], [155], [177], [16],[178]	RMSE	RMSE	RUL prediction	Temperature, pressure, flow
[110],[55]	Relative error rate	Relative error rate	RUL prediction	Stress or strain
[179], [113]	MSE	RMSE	RUL prediction	Current, voltage, temperature
[180]	F_norm	RMSE	RUL prediction	Capacities and voltage
[123]	MSE	RMSE	RUL prediction	Vibration
[181]	MSE	MSE, MAE, R2	RUL prediction	Vibration
[182]	Similarity distance	Prognostic horizon, $\alpha\lambda$ distribution, CRA, convergence, normalized RMSE	RUL prediction & Degradation prediction	Stress, crack length, pressure
[78], [123]	MAE, MSE	MAE, RMSE	Degradation prediction	Forces, vibrations and acoustic signal
[183]	RMSE	MAE	Degradation prediction	Vibration
[184]	MSE	RMSE	Degradation prediction	Stress
[114], [77]	MAE	MSE, test point-wise errors, relative error	Degradation prediction	Stress
[76]	Binary cross-entropy	F1-score	Degradation prediction	Cutting speed, temperature
[115], [185], [118]	MSE	MAPE	Degradation prediction	Stress or image
[116]	MSE	RMSE	Degradation prediction	Stress, viscosity, wind speed, and temperature
[43]	NMSE	NMAE	Degradation prediction	Spindle motor current
[186]	MSE	MSE	Degradation prediction	Far-field stress
[187]	Discretization error	MAE	Degradation prediction	Stress
[188]	RMSE	RMSE	Degradation prediction	Rise time, displacement
[93]	Negative log likelihood	Sensitivity analysis, MAE, and absolute error variance	Degradation prediction	Stress, temperature

2blueFrom Table 2, we can derive the following remarks:

1. Table 2 summarizes the training and testing metrics used in various PHM tasks for evaluating PIML models. Metrics such as Mean Square Error (MSE), Mean Absolute Error (MAE), Precision, Recall, F1-score, and others are employed to assess the performance of these models in condition monitoring, fault diagnostics, Remaining Useful Life (RUL) prediction, and degradation prediction tasks. MSE, MAE, and RMSE are the most commonly used training metrics in the collected literature, accounting for 35.7%, 20.0%, and 10% respectively.

2. In paper [76], the metric (binary cross-entropy), commonly employed for classification, is used for degradation prediction. This is because the prediction of degradation states is transformed into a classification of degradation levels.

3. One important aspect worth discussing is the embedding knowledge related to PIML models. In most studies, the choice of which type of knowledge to embed tends to be more based on subjective intentions. In practice, the knowledge embedded is strongly related to the monitoring signals used, e.g., the relationship between strain signals and deformation and damage growth, the relationship between temperature and fatigue, and the relationship between vibration and modalities, so collecting knowledge in this area from the available monitoring signals for use in informed machine learning would be a good place to start.

4. Table 2 also outlines the specific information about the corresponding monitoring signals utilized in each task, such as displacement, voltage, vibration, temperature, stress, current, and more. The vast majority of studies (94.3%) focus on processing time-series or one-dimensional monitoring signals, with only 5.7% of studies involving two-dimensional image signals.

5. The applications of PIML are mainly in the field of structures. The majority of processed signals in these applications are derived from vibration (25.7%) and stress (27.1%). In addition, certain non-destructive detection measurement methods, such as guided wave or acoustic emissions, are also utilized. Upon examining Table. 2, one can see that a wide array of metrics and monitoring signals are employed across various PIML studies. This diversity reflects the intricate and multidimensional nature of PHM tasks, highlighting the necessity for tailored approaches that align with specific applications and system characteristics. Gaining an understanding of the different combinations of physics knowledge, metrics, and signals utilized can serve as a valuable guide for researchers and practitioners when selecting appropriate evaluation measures and sensor inputs for their respective PHM applications.

3. Synthetic review of PIML studies in PHM

The physics involved in research subjects in the field of PHM are often diverse and can be expressed in different forms such as algebraic equations, differential equations, simulation results, logic rules, and probabilistic relations along with limited monitoring data. Therefore, it is necessary to provide a synthetic review of PIML studies in PHM from both perspectives: I) ***Expression forms of Knowledge*** and II) ***Knowledge Informed Methods***. 2blue Expression forms of Knowledge in Physics-Informed Machine Learning refer to the following knowledge expression forms:

1. Mathematical Equations: Knowledge is expressed through the formulation of mathematical equations that govern the underlying physics of the problem. These equations represent fundamental principles, physical

laws, and constraints relevant to the problem domain.

2. Conservation Laws: Knowledge about conservation principles such as mass, momentum, and energy conservation can be incorporated into physics-informed machine learning models. These laws provide important constraints that guide the learning process.

3. Differential Equations: Physics problems often involve differential equations that describe the relationships between variables. Expressing knowledge in the form of differential equations helps to enforce the physical behavior and relationships in the machine learning models.

4. Constitutive Relations: Knowledge about the material properties, constitutive equations, or parameterization specific to the problem domain can be incorporated. These relations provide insights into how different variables interact and influence each other.

Knowledge Informed Methods are considered as the different embedding ways of the usage of different knowledge expressions, the details are discussed in Subsection 3.2. Analyzing the knowledge expression forms involves examining how domain knowledge, physical laws, equations, and constraints are integrated into machine learning algorithms, providing insights into the underlying physics-ML convertibility. Knowledge-informed ways more specifically seek to leverage domain knowledge in designing and training machine learning models. However, it is crucial to also consider the broader perspective of analyzing knowledge expression forms, which is often overlooked in existing reviews. By analyzing both knowledge expression forms and knowledge-informed ways, researchers can gain a comprehensive understanding of the strengths and limitations of PIML approaches in PHM. This dual perspective enables a more rigorous assessment of methods and facilitates improvements in the design and implementation of physics-informed machine learning models.

3.1. Related review papers

To our knowledge, there is no meticulous review of PIML studies in the field of PHM but there are related works on the PIML topic. These works provide additional information that helps to get an overview of the PIML taxonomy as well as understand more about the research challenges on this topic.

The existing reviews, shown in Table. 3, argued that PIML is a promising solution to address the ML issues relating to physics consistency, data scarcity, and model interpretability, which are also valuable to PHM. They share a similar taxonomic view of PIML, describing that physics knowledge can be incorporated into data pre-processing, ML algorithm design, and regularization of the loss function.

Table 3: Existing review articles on PIML.

Authors	Topics of interest	Main Challenges	Taxonomy
Rai, Rahul, and Chandan K. Sahu.[72]	• Cyber-physical system's dynamic behavior modeling	• Discretization approximation of the continuous system behavior in a chaotic environment. • Scenario-oriented PIML hybrid framework. • Efficient extraction of causal and model parameter relationships in big data.	• Physics-based data preprocessing. • Physics-guided ML algorithm structure design. • Physics-based ML regularization item.
Willard, J., Jia, X.,Xu, S., Steinbach, M.[32]	• Engineering and environmental systems modeling. • Model solving methods.	• Embedding incomplete physics knowledge. • Keeping physical consistency in data mining. • Sparse data and uncertainty quantitative identification.	• Physics-based regularization item in ML algorithm. • Physics-guided ML initialization. • Physics-informed ML algorithm architecture design.
Kim, S. W., Kim, I., Lee, J., Lee, S.[189]	• Physics-informed deep learning in dynamical systems behavior modeling. • PHM is mentioned	• Designing prior informed deep learning framework. • ML training data scarcity. • Keeping physical consistency.	• Physics-informed Feature engineering. • Physics-informed NN structure. • Physics-informed loss function.
Jan Hagendorfer, Elias.[190]	Condition monitoring	• ML black-box nature explanation. • Training data scarcity. • Keeping physical consistency.	• Parallel/Series physics-ML combination structure. • Physics-based regularization item in ML objective function.
Finegan, D. P., Zhu, J.,Feng, et.al.[71]	Battery cell state prediction.	Keeping physical consistency.	• Physics-based data preprocessing. • Physics-guided ML algorithm architecture design. • Physics-based regularization item in ML algorithm.
Jianjing Zhang., Robert X. Gao.[191]	• Data curation and model interpretation for smart manufacturing. • PHM is mentioned.	• Non-interpretable prediction logic in deep learning. • Error or imbalance training data. • Data and data labels scarcity.	• Physical model bias compensation and unknown parameters estimation via deep learning. • Involving Physics-constraints into deep learning training .
Xu, Yanwen and Kohtz,e.t.al.[29]	• Reliability analysis and risk assessment. • Uncertainty quantification. • PHM is mentioned.	• Scenario-oriented PIML hybrid framework and its computational efficiency. • Incompleteness of physics knowledge and limited representatives of the training dataset.	• Physics-informed architecture • Physics-informed loss function
Thelen Adam, Zhang Xiaoge and Fink Olga.et.al.,[192, 193]	• Physical system modeling	• The need for accurate and reliable data to create an accurate digital twin model. • Integrating data from different sources and formats. • Selecting the appropriate modeling technique for a given physical system. • Scaling up the digital twin model to larger and more complex systems. • Validating the digital twin model against the physical system it represents.	• Modifying the loss function. • Generating synthetic data. • Pre-training on physics-based data. • Correcting models with unmodeled physics. • Correcting models with prediction residuals. • Learning to predict inputs.

2blue We greatly acknowledge the valuable perspectives and contributions presented in the existing reviews. However, it is important to note that these reviews tend to have specific disciplinary focuses, which may limit their comprehensive coverage of all critical tasks in PHM. Furthermore, while these reviews address the embedding of physics knowledge into ML approaches, they often lack a holistic analytical perspective throughout the entire ML process. Although they provide insights into how to incorporate physics knowledge, they do not fully explore the various sources of knowledge that can be utilized. Moreover, the existing reviews predominantly emphasize applications related to specific NN architectures, such as PINN, rather than embracing the broader framework of Physics-Informed Machine Learning (PIML). A recent study in provides a qualitative analysis and a comprehensive review of the role, taxonomy, and cases of PIML in the field of reliability [29]. PHM is part of the topics in the application Subsection. Our work complements their findings by providing a comprehensive quantitative analysis from the standpoint of knowledge in PHM, combining the complete qualitative analysis on the most advanced researches. Additionally, we not only review taxonomic and informed methodology but also examine the various forms and sources of informative knowledge. Besides, the studies in [192, 193] provide a systematic review of hybrid modeling in digital twins and briefly discuss the significance of PIML technology. However, these studies primarily focus on the analysis of modeling system responses and dynamic behaviors in the context of digital twins, and only partly include the qualitative and quantitative aspects of PIML in the specific context of PHM. Our paper, on the other hand, is specifically dedicated to PHM delving much deeper into these aspects, while of course not being as exhaustive in terms of the other aspects of digital twins. Considering the limitations of the existing papers, this review aims to address the gaps in the state of the art by providing a more thorough and analytical perspective on PIML methods within the realm of PHM. By integrating both qualitative and quantitative approaches, our research endeavors to contribute to a holistic comprehension of PHM and its practical applications. Furthermore, it aims to elucidate a broader understanding of the entire machine learning process, encompassing all critical tasks involved in the integration of physics-based knowledge.

3.2. Taxonomy of PIML in PHM

Depending on the role of physics knowledge and its informed position in the hybrid model, we propose to classify PIML methods into three categories. The first category uses physics knowledge to guide the construction of the input space, i.e., “Physics-informed inputspace”. The second category named “Physics-embedded algorithm structure” 2blueincorporates physics knowledge into the model architecture in machine learning process. The third category embeds Physics-constraints on the ML objective function to conduct “Physics-constrained learning”. These three categories correspond to three typical solutions to ML problems: input data optimization, model architecture optimization, and objective function optimization. Based on the combined roles of physics knowledge in different parts of the ML pipeline, we have summarized the 8 types of informed patterns, including “Simulator”, “Gauge”, “Extractor”, “Operator”, “2blueStructure blueprint”, “Initializer”, “Consistency check”, and “Conflict check”, as shown in Fig. 10, which covers all aspects of ML data flow. Their corresponding implementations, and the related ML technical frameworks for achieving these implementations are also summarized. It can be seen that NN are the most widely used modeling tool. 2blueTo assist in the understanding of the methodology, the same knowledge with different informed ways are shown in https://github.com/pimlphm/Physics-informed-machine-learning-based-on-TCN.

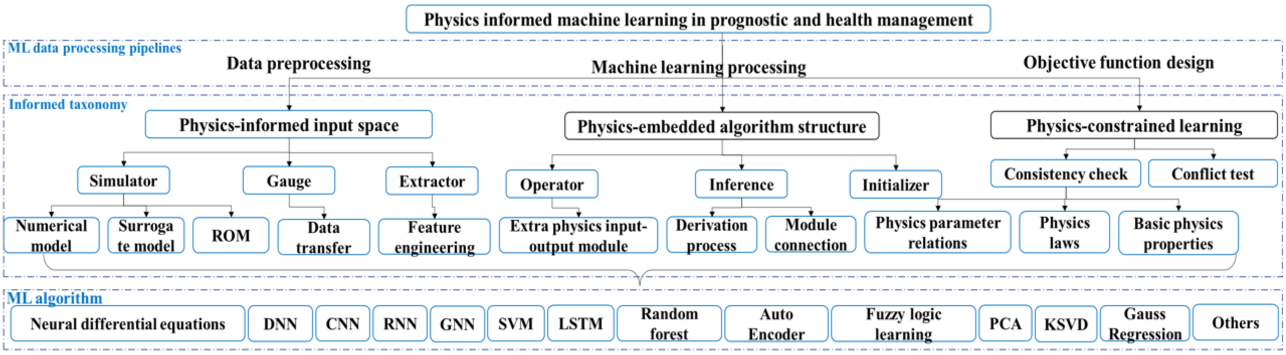


Figure 10: Taxonomy of existing PIML methods in PHM

3.2.1. *Physics-informed input space*

Data preparation generally occupies the most workload in PHM [194]. Regarding the category "Physics-informed input space", PIML seeks to gain physics information in the ML input space, distilling the multi-sources and heterogeneous monitoring data [45, 46] by assisting data augmentation, feature transformation, feature selection, dimensionality reduction [155], and information fusion [40]. "Physics-informed input space" can be seen as an extension of the traditional "feature engineering" or "simulation-based data augmentation" processes by using physics knowledge to drive data processing and augmentation, including three paradigms: "Simulator", "Gauge", and "Extractor", which are shown in Fig. 11.

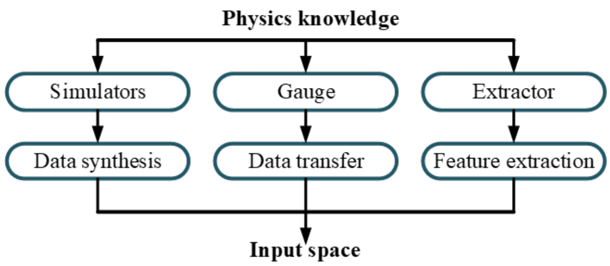


Figure 11: Three ways to construct a physics-informed input space.

The technologies "Simulator" and "Gauge", which occur in the "Data preparation" step, aim to generate and transform data. Meanwhile, the "Extractor" in the "Data preprocessing" step is dedicated to extracting useful features. A brief summary of these three technologies in the existing works is shown in Table. 4.

Table 4: Summary of physics-informed input space studies in PHM.

Ref.	Application	Knowledge source	Informed	ML framework	PHM tasks

[195, 196]	Aeronautical structure	Component-based digital twin	Simulator	Classification tree and SVM	Fault diagnostic
[170]	Triplex pump	Component-based digital twin	Simulator	Auto-encoder transfer learning	Fault diagnostic
[58]	Oil production-line	Production-based digital twin model	Simulator	Autoencoder & LSTM	Condition monitoring
[197]	Rotor	A priori evaluation of feature space separability of loads	Simulator	Hamiltonian autoencoder NN, PCA, & random forest	Fault diagnostic
[188]	Electro-Hydrostatic Actuator degradation	Physical degradation model	Simulator	LSTM	Degradation prediction
[13]	Tubofan engine	Engine air path performance model	Simulator	DNN	RUL prediction
[55, 53]	Composite structure	Bonded joints fatigue FE or lattice surrogate model	Simulator	FCN	Fatigue prediction
[198]	Bearing	Time domain statistical feature generation model	Simulator	SVM	Fault diagnostic
[174]	Aircraft composite structure	A numerical solutions of Lamb waves	Simulator	CNN	Fault diagnostic
[144]	Industrial production	Time-series derivative weighting for perturbation values	Simulator	VAE	Fault diagnostic
[199]	Building	Invariable characteristics of building structure	Gauge	Physics-informed multi-source domain adversarial networks	Fault diagnostic
[59]	Additive manufacturing monitoring	Geometry invariant in thermal history features and trend	Gauge	Tree-based regression	Condition monitoring
[171]	Gearbox	Implicit physical association between unlabeled and labeled data	Gauge	Deep convolutional generative adversarial network	Fault diagnostic
[200]	Gearbox	Vibration inherent cyclostationary characteristics	Extractor	Autoencoder	Fault diagnostic
[183]	Bandsaw	Vibration modal analysis and finite element analysis	Extractor	PINN and DCNN	Fault diagnostic
[167]	Gearbox	Health-adaptive physics time-scale representation embeded input module	Extractor	CNN	Fault diagnostic
[201]	Electro-mechanical load	Feature space load separability prior evaluating	Extractor	SVM & DNN	Fault diagnostic
[169]	Air handling units	Importance feature selection based on the semantics of the physical model	Extractor	isserstein generative adversarial network	Fault diagnostic

Physics-informed simulators

The works in this group focus on the construction of simulators that capture the physical behaviors of the studied system to generate data for training ML models. The data generated by those simulators provides richer information that covers different health states of the system and reduce the knowledge blindness of ML and thus enhances ML performance. 2blueTo construct the simulator, various models with different degrees of simulation can be exploited such as structure-based and process-based digital twin models[192, 193], engine performance models, or components' finite element models. The challenge when implementing a physics-informed simulator is to find a balance between simulation accuracy and speed. Its basic paradigms is shown in Fig. 12.

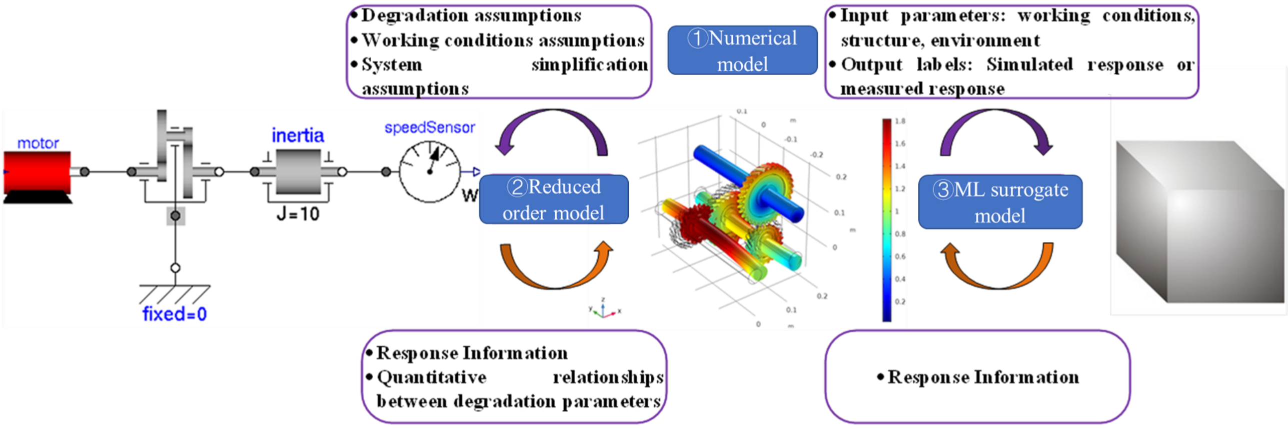


Figure 12: Three ways to construct a simulator for physics-informed input space.

Traditionally, to construct a physics-informed simulator, the numerical model's output is used as the input of the ML model. However, high-fidelity simulations are computationally costly. Therefore, most research has focused on the use of a reduced order model (ROM) or a surrogate model to lower the simulation cost. The former are the simplifications of huge scale models for establishing an approximate description of multidimensional physical processes in low dimensions. The combination of ROM and virtual sensors can create dynamic model calibration [58], which is actually the basis of many simulation software (e.g., Ansys and Modelica). Digital twin-based physics-informed input models in the collected literature also augment the input space of ML by simulating certain types of physical signals based on a ROM of the system's specific behavior. Besides, surrogate models compute the response of the original high-fidelity model at a chosen finite number of points. In fact, it is a proxy for the real system at finite operating states [202]. In existing PIML methods, building ROMs usually reduces the amount of unquantified variables in the model by adding constraints. This increases the preconditions for device operation and specifies the state space involved, which relies on the user's understanding of a specific failure. In contrast, surrogate models in existing PIML studies tend to complete the modeling by fitting a ML model to the relationship between operating conditions and system response under finite operating conditions. For example, in the simulation of the meshing vibration behavior of a gear train, the ROM-based PIML simulator specifies the operating conditions of the gear train (load, speed, etc.). It uses a simplified physical model (e.g. a time-varying stiffness spring-damping model for meshing gear pairs) to represent the components and a simplified data flow connection to represent the shaft structure. It models the system mass as a centralized inertia module, ignoring the non-linear coupling, electromechanical coupling, and changes in dynamic states under different operating conditions. The effect of tooth wear and tooth breakage on the meshing stiffness are the only factors to be considered. In contrast, the PIML simulator based on the surrogate model uses a grey box model to fit the relationship among the excitation, responses, and structural parameters. It assumes that the corresponding mappings of the grey box model are constant and applicable in different working state spaces, thus further increasing the number of samples.

Physics-informed gauges

In some specific case studies, e.g., complex structural systems, it is inevitable to use simplified physical models for the construction of "Simulators". However, this simplification might lead to significant deviations in model behavior as well as in estimated values compared to the true values of the system [80]. Then, model updates can not inherently correct modeling errors. To overcome this issue, data transferring is an alternative solution for enhancing the data space. In this light, some studies focus on applying physics knowledge as a "Gauge" to evaluate the similarity between the source and target database. This technique migrates feature knowledge from the source domain to the target domain by designing a physically based transfer criterion between them. It allows enhancing the ML robustness and improving the efficiency and accuracy of ML models.

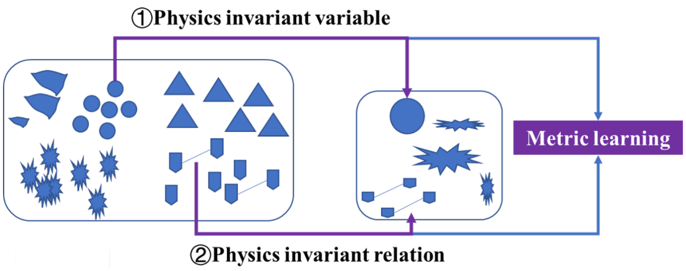


Figure 13: Two ways to construct a gauge for physics similarity metric-informed learning.

The two basic ways to implement "Gauge" are shown in Fig. 13. Its principle consists of finding the invariant variable or invariant relationship between the source and target domains, such as feature symmetry, conservation, transformation invariance, and monotonicity [139, 203]. The source domain, which has a large amount of data and information, is then selectively transferred to the target domain according to physical similarity criteria [159].

Physics-informed extractor

In addition to data augmentation, data processing is another crucial task that directly affects the performance of ML models. To ensure that the input space contains as many fault-related features as possible, it is necessary to create a physics-informed extractor to guide the data preprocessing according to physics knowledge. For example,

the proposed physics-informed extractors allow selecting suitable domain transformation methods [204] to get the relevant aggregated features [155], or fusing heterogeneous information from multiple sources [40, 65]. In [88], the taut string model equation standardizes the principal component analysis method for extracting the specified modal frequency bands of cable vibration. The study in [79] develops a physics-guided ML model to conduct the feature extraction process that can generate particular features directly reflecting the performance of electric vehicles.
2blueFollowing feature extraction, the ML module incorporates an embedding component that functions as a set of sub-feature extractors. Subsequently, information fusion takes place, with a primary focus on merging physical health indicators with virtual health indicators. The former pertains to fault physics and typically carries significant interpretability in terms of the indicators. For the latter, there are two implementations in existing PIML studies[205]:

- Information fusion from multiple physics domains to obtain "**sensory data**" with less redundancy and representing all original information. For example, multiple regression [132], elevated space projection [206], and other supervised and unsupervised learning methods are used to perform signal-level data fusion and feature-level data fusion [42].
- Cross-physics domain relations fusion through physics relationships to get "**perceptual data**", where the physics relationships of the various parts of the data are prominent [207, 208], as shown in Fig. 14. For example, in crack growth prediction, information on the structural response, such as displacement and phase fields, obtained by the Newton-Raphson solution, are preserved in the form of images of the current state of the crack to build spatial structural knowledge [70].

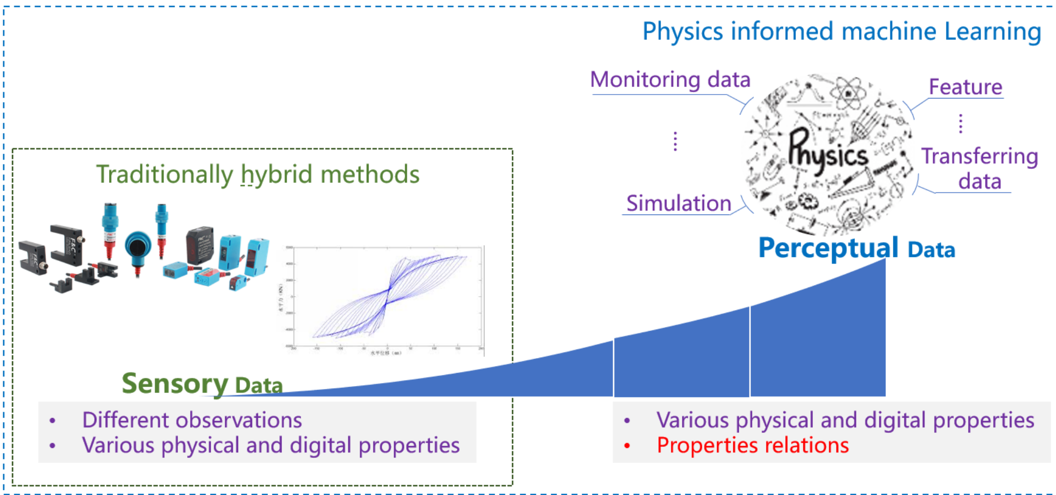


Figure 14: PIML based information fusion: Extending and distilling multiple heterogeneous sensory signals into perceptual signal.

3.2.2. *Physics-embedded algorithm structure*

Regarding "Physics-embedded algorithm structure", PIML seeks to make the traditional physics-agnostic ML become physics aware so that the governing processes are added to the design of ML algorithm structures and the parameters searching process. It is prone to integrate the "Hard Constraint Projections (HCP)" [90] with ML, including the three following paradigms: "Basic operator", "ML 2blueStructure blueprint", and "Parameter initializer", as shown in Fig. 15.

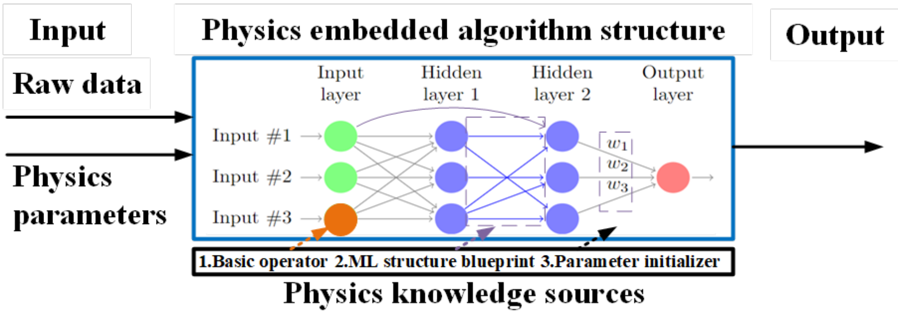


Figure 15: Three ways to construct a physics-embedded algorithm structure. 2blueThe different colored lines represent different implementations, orange arrow represents embedding physical knowledge into a local module of ML such as a neuron, purple arrow is designing inter-module connections such as layer connections based on physical knowledge, black arrow represents initializing ML parameters

2blue The "Basic Operator" is responsible for enforcing physically resolved relationships in machine learning processing. On the other hand, the "ML Structure Blueprint" is dedicated to designing ML modules or inter-layer connections based on physically derived relationships, thereby endowing sparsity. These components are implemented in the algorithm's structural design. Additionally, the "Parameter Initializer" focuses on identifying the ML parameters. A brief summary of these three approaches used in the existing literature is shown in Table 5.

Table 5: Summary of the studies on physics-embedded algorithm structure in PHM.

Ref.	Application	Knowledge source	Informed	ML framework	PHM tasks
[182]	Crack growth and filter clogging	Paris laws for fatigue crack and pressure drop analog formula	Operator	ANN	RUL prediction
[165]	Motor bar broken	Fault frequency and square envelope threshold	Operator	CNN	Fault diagnostic
[209]	Drill pipe	Embedding hydraulic coefficient relationship between two DNNs	Operator	DNN	
[113]	Lithium-ion battery battery	Reduced-order model based on Nernst and Butler–Volmer equations	Operator	RNN	RUL prediction
[43]	Tool wear	Sipos empirical wear-time	Operator	Adaptive neuro-fuzzy inference system	Degradation prediction
[81, 130]	Material defect	Topology of wave-guided electromagnetic acoustic sensor systems	Operator	Siamese CNN	Fault diagnostic
[149]	Bearing fatigue	Paris-laws based corrosion	Operator	NN	
[114, 129]	Structure crack	Damage differential equations & Dirichlet boundary based growth laws	2blueStructure blueprint	DeepONet	Degradation prediction

[37, 210]	Crack identification	Differential equation for crack extension	2blueStructure blueprint	Stacked auto-encoder	Degradation prediction
[121, 115, 186, 179]	Aviation structure crack	Crack extension or vibration anomaly models	2blueStructure blueprint	RNN	Degradation prediction
[91, 120, 115]	Structure fatigue	Eulerian integration for fatigue crack extension	2blueStructure blueprint	RNN or CNN	Degradation prediction
[113]	Batteries RUL prediction	Governing differential equations based on measured capacity & voltage curves	2blueStructure blueprint	RNN	RUL prediction
[156, 178]	Structure damage	Structural changes due to damages	2blueStructure blueprint	Stacked NODE	Fault diagnostic
[95, 166, 211, 212, 20]	Grid and Buses FD	Physics spatial or spectrum associativity	2blueStructure blueprint	Graph NN	Fault diagnostic
[154]	DRAM error	Spatial dependence of the DRAM	Initializer	SVM, NN, Boosted Trees, Naive Bayes, Random forest	Fault diagnostic
[168]	Bearing	Interpretable weights based envelope spectrum	Initializer	Supervised learning dichotomy	Fault diagnostic
[57]	Casting defect	One-dimensional heat transfer equation	Initializer	Non-negative matrix factorization	Condition monitoring
[142]	Materials cracks/fractures	Geomechanical alteration index cluster basis	Initializer	K-Means cluster	Fault diagnostic
[76]	Tool wear	Decision space parameterized by cutting speed and temperature	Initializer	CNN	Degradation prediction
[149]	Power grids	Wind oscillation equations and grid equations	Initializer	Gaussian Process Regression	Fault diagnostic
[187]	Offshore wind turbine	Degradation excess matrix	Initializer	Bayesian network	Degradation prediction

Physics-informed operator

The principle of the “Operator” is to use physics-knowledge of failure mechanism to build ML modules that allow better capturing input-output relationships. To do this, there are two ways proposed by the existing studies: 1) Replacing ML modules with physical input-output models, 2) Custom layer and neuron to express physics equation, as shown in Fig. 16.

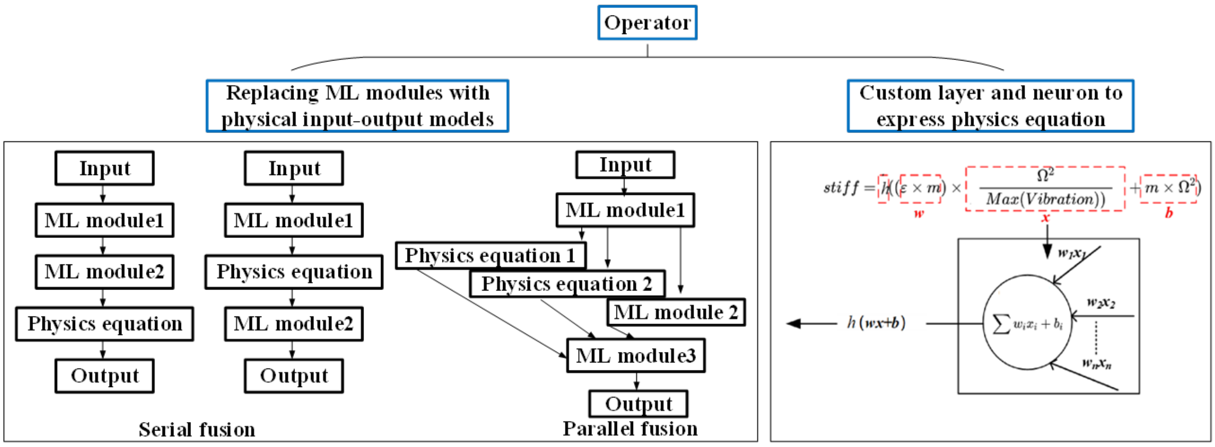


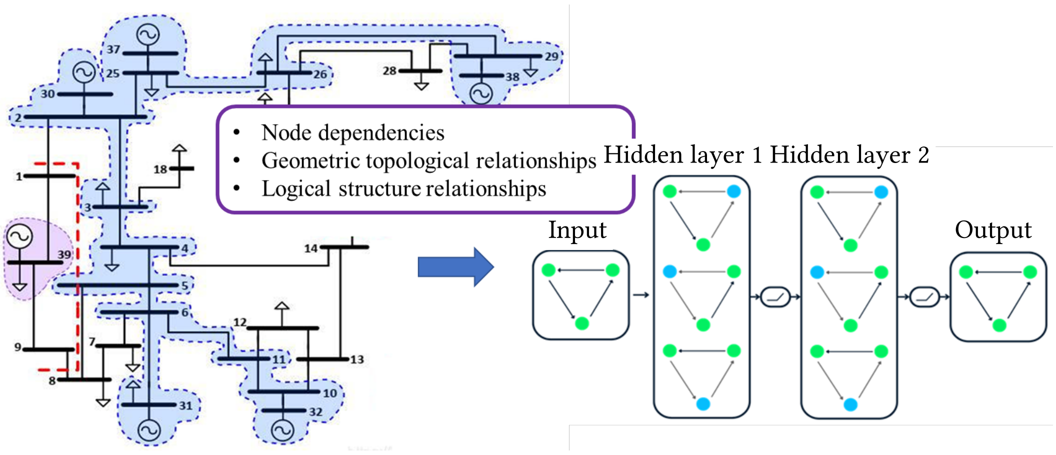
Figure 16: Two ways for embedding physics knowledge in the form of an operator.

2blueThe first approach, *replacing ML modules with physical input-output models*, performs a physically meaningful transformation of the raw data into health indicators required by the subsequent ML modules. Then through integrating ML modules for a fusion of information across physics models and ML modules. For example, in the papers [172, 213], the customized wavelet transformation layers are designed to guide the feature extraction and health indicator construction tasks by assigning the appropriate coefficients and weights for NN layers. The overall structure includes both series and parallel fusion methods for the output from the physical embedding part and the output from the ML module processing [116]. The serial architecture selects the best method for each data characterization and decision-making step. Compatibility between successive methods is crucial for sequential re-evaluation of previous outputs, reducing ambiguity and improving accuracy. However, accumulating errors from incomplete physics information is a potential drawback. The parallel fusion structure combining physics models and machine learning (ML) modules offer the advantage of leveraging the strengths of both approaches simultaneously. It plays the role of the compensator in enhancing accuracy, robustness, and interpretability while enabling a comprehensive understanding of complex systems' degradation behavior [179]. However, challenges include complexity, data requirements, compromised interpretability, potential conflicts between models and algorithms, and the need for expertise and resources in development and maintenance.

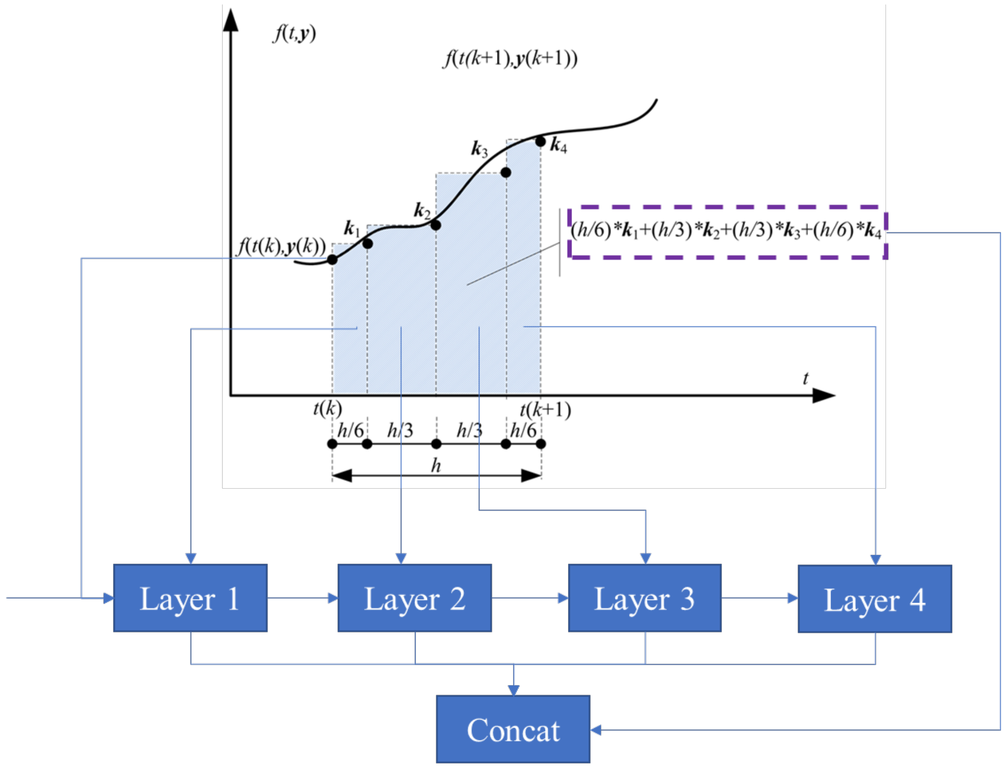
In the second approach of using ML modules to express physical functions, the ML module acts as a forcing actuator to derive the physics model output and provide additional physical information. This approach involves utilizing mathematical approximations through the ML's intrinsic functions. For instance, in the paper [181], a linear summation of a NN is employed to represent the relationship between vibration amplitude and rotational speed. The activation function and connections between NN layers approximate the relation between features and RUL values. Trainable weights, biases, and nonlinear activation functions represent the unknowns and parameters in the formula.

Physics-informed ML 2blueStructure blueprint

Compared to the "PIML operator", "PIML 2blueStructure blueprint" is a physics-aware method that focuses more on guiding the building of data flow similarity between ML and physics knowledge, including modeling the physical processes, measurement processes, derivation processes, geometric structures, and so on, as shown in Fig. 17.



(a)



(b)

Figure 17: Building PIMLs with similar derivation processes or structures to PBMs.

2blueDue to the fact that the essence of this class of methods lies in designing the structure of machine learning, including module design and inter-module connections, we propose to use “Structure blueprint” to represent physical reasoning processes or physical structural relationships informed machine learning algorithm structure. It aims to find topological similarities and the unit dependencies mappings from the geometric structure, system behavior, or internal material interaction [101] to ensure the physical priority of the reasoning process when training ML models. The conjecture and the abstraction of the system behavior in PBMs are useful sources to optimally guide the training process of ML. For example, the NN is designed according to the topology and physical laws of an electric grid in [95]. The underlying physical model governs the operation of the distribution network to sparse the learning model’s structure where the pruning is done in a deterministic manner during the training process [132]. As a result, load anomalies and grid damage can be indicated by changes in the output of the network nodes. Besides, in reference [102], the NN gradient models a potential energy function that is exploited to represent the dependency of the interference between quad-copters and their distance.

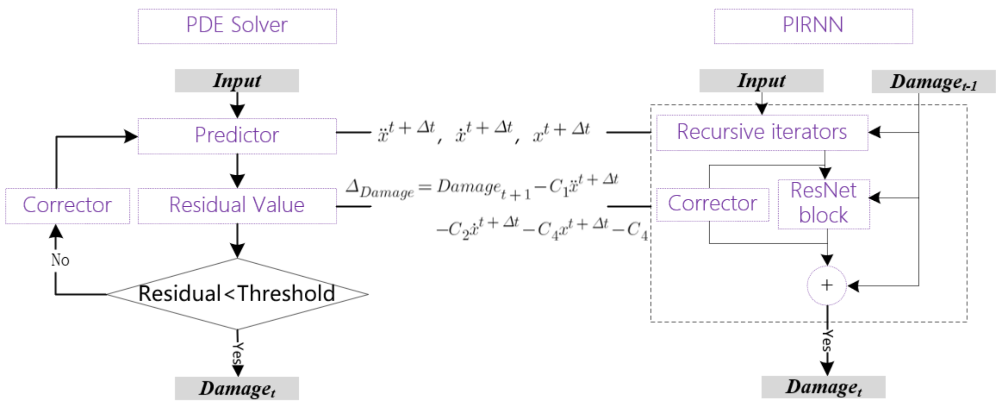


Figure 18: Building PIMLs with similar derivation processes or structures to PBMs.

Several special structures can be utilized to model physically derived relationships. In Fig. 17b, each step of a Runge-Kutta numerical integration process is represented by a NN layer, and the integration path calculation is completed based on the physical summation relation. The dynamic behavior changes, such as damage growth in RNN [120], are expressed through a recursive prediction structure (see Fig. 18). Each formula within the recursive relationship for damage growth is represented by a custom NN layer, and the inter-layer structure enables the realization of the recursive relationship. In representing the derivation process in terms of structure, much physics knowledge is further de-analyzed [50]. For example, in the paper [37, 156, 178], the specific physical relationships are non-analytical. The embedding of physics knowledge is accomplished by two NN sharing parameters in a CODEC (Coder-Decoder) structure. The latter is a proxy for the linear second-order partial differential equation for acoustic wave propagation, while the former is used to approximate the solution of the model from the measurements to the latter surrogate

NN. In summary, "PIML 2blueStructure blueprint" defines ML reasoning process as part of the physics derivation form where the ML modules retain their original computational structure, but acts as a mapping of certain types of the physics equations solving process by constraining the inter-module relationships.

Physics-informed ML parameters initializers

Unlike the focus on both "PIML operator" and "PIML 2blueStructure blueprint", the research on "PIML parameters initializers" is more concerned with the selection and assignment of ML parameters and hyper parameters. For example, the weight selection is implemented based on physical energy minimum state completion in Markov random fields (MRF) [154]. In [36], wavelet-based features of the multi-scale envelope spectrum are fused by a statistical health index generating model, and the observation function between the defect state and the fused features is assumed to be a linear fitting. The empirical model for a spalling propagation based on the Paris formulation is a predictive model for which the initial parameters are set as probability distributions. Besides, the average value of the one-dimensional heat transfer equation solution is used as the initial parameter for the factorization of the non-negative matrix for casting defect monitoring [57]. In summary, the initialization of ML parameters in these studies is usually based on the physical model solution.

3.2.3. Physics-constrained learning

In contrast to the hard constraints of "Physics-embedded algorithm structure", PIML also includes soft constraints that enable ML to produce an approximate satisfaction of a given set of physics through the design of the objective function. Its approximate satisfaction can be introduced in the form of integration, differentiation, probability, logic rules, and other forms of physics-based deviations. According to the relationship between the informed physics objective function and the original ML objective function, this paper groups "Physics-constrained learning" into two paradigms: "consistency check", and "conflict test", which are shown in Fig. 19.

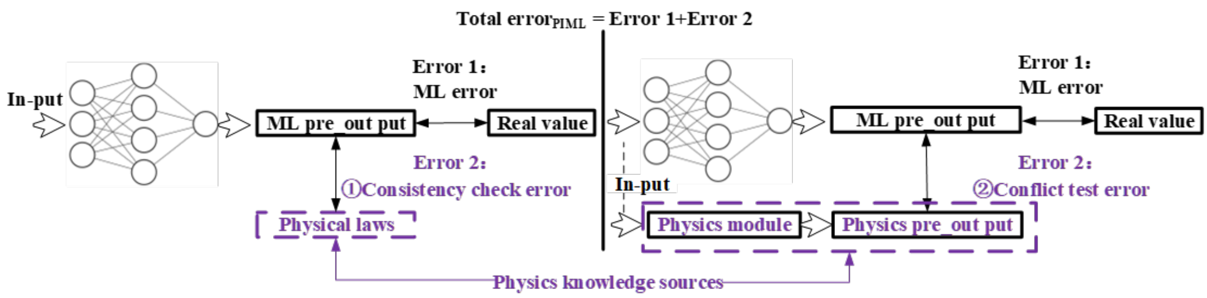


Figure 19: Two ways to construct physics-constrained learning.

The total error of the PIML model includes a traditional ML prediction error ("Error1") and a physical consistency error ("Error2"). In general, the numerical best fit to the available data (residual loss) and the consistent satisfaction of physics principles (boundary loss) show discrepancies [147]. Designing an objective function based on "consistency check", or "conflict test" error is dedicated to the convergence of ML results towards physical consistency. The related literature is summarized in Table. 6

Table 6: Summary of Physics-constraint learning in PHM.

Ref.	Application	Knowledge source	Informed	ML framework	PHM tasks
[177]	Turbo engine	Loss based on PDE residuals	Consistency	Stacked CNN	RUL prediction
[96]	Deformation identification	Normalized physics model's modal residual	Consistency	DNN	Fault diagnostic
[133]	Material damage	Finite Element Analysis	Consistency	DNN	Fault diagnostic
[123]	Bearing	Reliability model based on Weibull	Consistency	ANN	Fault diagnostic
[214]	Vehicle sensor	Residue generation based on transferable operators	Consistency	Neyman-Pearson test	Fault diagnostic
[132]	High impedance fault detection	Elliptic equation of rotational trajectories of the voltages and currents	Consistency	Autoencoder	Fault diagnostic
[215]	Building	Attribute-category matrix	Consistency	MatConvNet	Fault diagnostic
[146]	Ocean current turbine	Characteristics in frequency domain of the mean water flow velocity in the fan balance	Consistency	PCA and CNN	Fault diagnostic
[150]	Metal damage	Atomic update based on the regularization term of the one-dimensional wave equation	Consistency	K-SVD	Fault diagnostic
[131]	Workshop machinery	Fault frequency domain feature loss related Pearson correlation coefficient	Consistency	Deep convolutional autoencoders	Fault diagnostic
[176]	Damage stress prediction	FEM based stress distribution	Conflict	LSTM	Fault diagnostic
[143]	Bearing	Expert experience-based fault degree threshold model	Conflict	CNN	Fault diagnostic
[84]	Steel building damage	Output of a finite element model	Conflict	DNN	Fault diagnostic
[126, 173]	Wind farm & gas turbine	Physically complete historical dataset	Conflict	ANN	Fault diagnostic

From Table. 6, one can see that physics knowledge is used directly in the ML target design by modifying the target function in such a way as to influence the parameter changes during the ML optimization-seeking learning process. For specific PIML frameworks, the "consistency loss" strives to ensure that the ML output conforms to the physical fact, while the "conflict loss" is built by the conflicts between the ML output and the physical model output.

Consistency loss design

In tool wear prediction [78], the empirical knowledge (that wear increase as the number of cuts increases) is then compiled into a function that detects trend information in the output sequence. In the case of ultrasonic detection of damage to metal sheets, consistency is expressed in the ability of the algorithm to identify results that are close to the analytical model corresponding to the damage cluster and satisfy the regular term generated by the residual from the governing equation [37]. In the K-SVD method for metal damage identification, the article [150] builds an ultra-complete dictionary with an additional one-dimensional wave equation-based regularization term for the atomic update process of the dictionary.

In these studies, the output of the ML needs to satisfy the regular term or lower the punishment function value of the governing equation for physical consistency, in addition to the original fitting accuracy as possible. And this governing equation can be a partial derivative relation that represents an approximation [37]. It can also be whether certain explicit physical equations are met within the required tolerances. For example, the rotational trajectories of the voltage and current need to satisfy the elliptic equation in autoencoder-based high impedance fault detection [132]. In some cases, the design of consistency loss does not require a fully known analytical physics model. It is equally feasible to enforce the differential equations through a NN as a trial solution to the degradation differential equations and through additional iterative pathways outside the NN [133].

Conflict loss design

Based on the inconsistency between the physics model and the ML output, it is also possible to design a "conflict test", which only optimizes the relevant parameters of the ML in the error propagation process.

In [143], the results of a diagnostic conflict based on an artificial fault threshold model with a deep CNN are used to design the loss function that aims to improve the discrimination of the severity of bearing faults. A physics-based loss function is designed to evaluate the difference between the output of a NN model and the output of a finite element model update in steel building damage detection [84]. This idea can be seen as a traditional fusion approach, which combines the outputs of different approaches [216] in the ML training process. The main difference between them is that the physics-based outputs are used here primarily to correct the behavior of ML rather than enhance decision-making.

4. Discussion of PIML studies in PHM according to the form of physic knowledge

The previous sections summarized the different PIML frameworks in PHM. They initially answer the question of "how to inform physics knowledge in ML". However, physics knowledge is an extremely complex abstract concept, and the question of "what kind of physics knowledge can be used for informing ML" has not been addressed yet. This question is then considered in Subsection. 4.1. Next, Subsection. 4.2 is intended to synthesize the informed way of that knowledge in literature.

4.1. Physics knowledge categories

Physics knowledge is the prerequisite for implementing PIML. In review [141], the authors propose categorizing the knowledge sources according to their origin. However, the PIML implementation methods depend on the form of knowledge rather than the source of knowledge. For example, the proposed PIML frameworks in papers [84] and [133] come from different fields (building construction and material industry) with different knowledge sources, but both of them use the same knowledge form, i.e., finite element methods, to build the "consistency check" loss function. Therefore, this Subsection focuses on synthesizing the form of physics knowledge instead of its source. From the existing studies on PIML in PHM, the physics knowledge forms can be classified into three categories, as shown in Fig. 20. **1) First category: *Explicit knowledge related to analytical failure models.*** The explicit

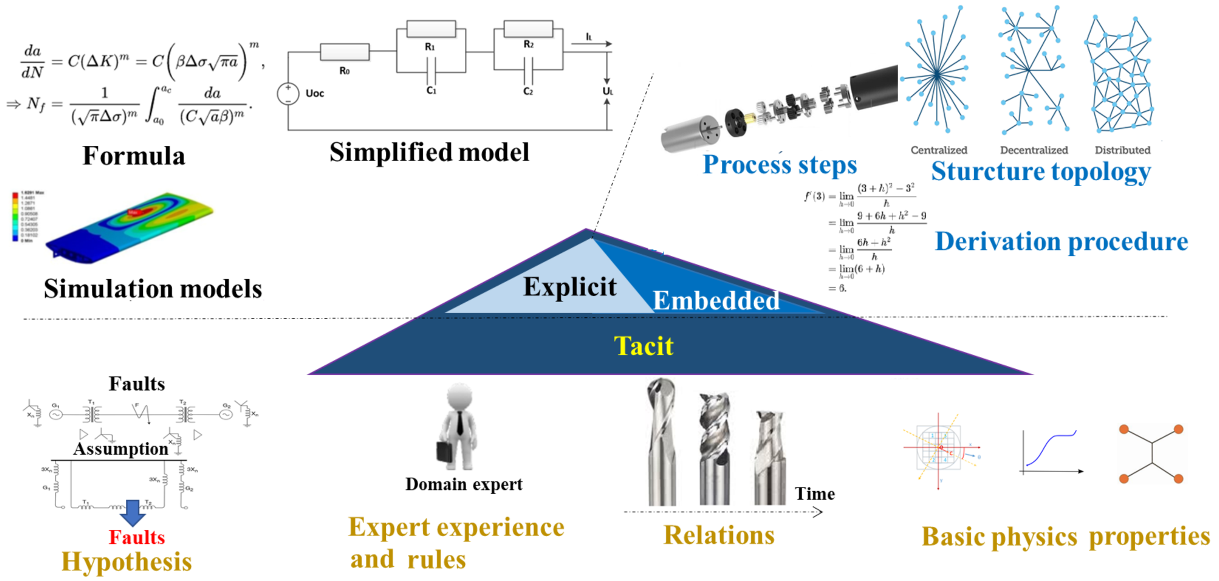


Figure 20: Physics knowledge forms present in existing studies of PIML in PHM.

knowledge is represented by analytical models or equations of system dynamic behaviors, such as generator inertia constants, damping coefficients, and rotating speed in rotor dynamics [140]. They are mathematically and physically unambiguous, formal, symbolic and structured. Particularly, in the field of PHM, they demonstrate the quantifiability of the failure processes, including algebraic, governing equations, and probabilistic relations.

2) Second category: *Embeded knowledge related to a structure or specific process*. It is locked into the physics derivation process, system convention, structure, or layout. It provides information related to the sequence orders and the requirements of each process step or each component structure. It uses ML modules to express information concerning the system structure [132], the unit dependencies [154], or the system topology framework. In particular, some knowledge is non-symbolic and non-explicit, being merely an input-output or mutual verification relationship between the derivation procedures. **3) Third category: *Tacit knowledge relating wide range of physical information*.** It involves hypotheses, expert rules and experiences, and also diverse underlying physical properties. It refers to knowledge about the deterioration process which is somewhat intuitive and difficult to quantify.

4.2. Discussion of physics-informed ways according to knowledge forms

Table. 7 summarises different forms of knowledge for PIML in PHM and presents their corresponding embedded way into ML. From this table, we can note that:

Table 7: Summary of the knowledge-informed ways according to the knowledge forms.

Ref.	Knowledge forms			Informed ways
	Explicit	Embeded	Tacit	
[195], [196], [170], [58], [197], [188], [13], [53], [55], [198], [174], [144]	✓			Simulator
[199], [59], [171],			✓	Gauge
[199], [59], [171], [200], [183], [167], [201], [169], [169]	✓			Extractor
[182], [36], [62], [172, 213, 63], [113], [149]	✓			Operator
[118], [43], [165], [51]			✓	Operator
[81], [130], [209]		✓		Operator
[37], [210], [113], [217]	✓			2blueStructure blueprint
[114], [129], [121], [115], [186], [179], [50], [120], [91], [114] [156], [178] [95], [166], [211], [212], [20], [107]		✓		2blueStructure blueprint
[57], [142], [76], [149], [187]	✓			Initializer
[154], [168]		✓		Initializer
[96] , [133], [123] [132], [150]	✓			Consistency
[177], [214]		✓		Consistency
[146], [131]			✓	Consistency
[176], [84]	✓			Conflict
[143], [126] , [173]			✓	Conflict

1. Due to explicit analytical equations or models that define clear input-output mathematical relationships, explicit knowledge is the most common way for building PIML. It is widely used in the construction of "simulators", "extractors", "operators", and "consistency checks". It can often be used independently or collaboratively in several data flow sessions in a ML pipeline. It changes the input and output of the corresponding link on the data stream but does not change the data flow direction.

2. 2blueEmbedded knowledge studies focus on serving as the physics-informed ML structure design guidance. They seek to build the entire ML structure such that the information flow inferred through the ML model

resembles the one passed through a real physical model, structure, or derivation process. In typical circumstances, when there are unknown terms in the process of physical derivation, making it difficult to establish a formulaic model, and when there exists a quantifiable relationship between physical structures, one may employ paradigms such as 2blueStructure blueprint (Designing the structure and parameters of ML model solely based on physical structural relationships or deductive processes), or alternatively, embedding limited known steps or models as local operators within the ML framework.

3. A large part of "Embedded knowledge" studies actually points to interchangeability between ML and physical derivations. For example, the Eulerian solution of Ordinary Differential Equations (ODE) is implemented as a special case of RNN which is applied in Dynet [218]. Although the current research still focuses on the relevant area of neural differential equations (NDEs), i.e. the use of ML to derive or embed the differential equation for failure as in the paper [217, 156, 178], the trend of operator learning, led by Deeponet, has recently gotten a lot of attention. For illustration, the transmissible operators, which characterize the relationship between the outputs of an underlying vehicle sensor system, have been proposed in the paper[217].

4. Although "Tacit knowledge" is the most widespread knowledge, only a small number of studies have been conducted on it. In these studies, "Tacit knowledge" is usually transformed into a parsable form in order to be embedded in the objective function and derivation process. It enables the design of a physics similarity test metric [199, 171] to assess whether the distribution or trend of results conforms to certain physical properties [177, 214], as well as the construction of a conflict loss [143, 126, 173].

5. For the use of tacit knowledge, the physics knowledge is often not given in advance, but it is obtained by designing a ML model in a reasonable way to discover the fault-related information. For example, in [118] the authors use dynamic mode decomposition to extract signal characteristics. These characteristics are used as labels for training the ML on how to automatically discover the information related to crack growth. The implementation of this type of knowledge discovery process should lie in stacked ML architectures, i.e., one ML model for knowledge discovery and one for proofreading or extending knowledge. For illustration, one can consult the associated two-stage graph NN architecture in [132].

6. In practice, analytical and quantifiable explicit knowledge is certainly restricted, and knowledge regarding fault processes is still largely perceptual or qualitative. Hence, tacit knowledge can be transformed into embedded knowledge through a deeper understanding of mechanisms and structures. For instance, the node and connection in the graphical NN can be constructed based on an understanding of the current and voltage distribution in the electrical grids [149]. Furthermore, the understanding of the basic physics properties and relations can be described in terms of a formula and translated into explicit knowledge. For illustration, an embedded transfer learning model based on the physical attributes of buildings' damage patterns is trained by minimizing the loss of the damage attribute that is measured via L2-norm and angular loss [215]. Besides,

the aforementioned building-related knowledge can also be used to introduce a new physics guided weighted design. In [199], the authors use physical similarity to the target to measure the importance of each source and thus decide the data of which source to transfer.

7. The same physics knowledge can be informed in different ways. For example, dynamic mode decomposition capturing system characteristics can be used to design operators for image reconstruction to identify cracks [118], or to design an extractor that generates input feature maps for time-delay-system diagnostic [169]. In particular, knowledge in the form of self-contained input-output relationships and derivations such as finite elements can be used as 1) embedded knowledge to guide the network design for simulating the physics derivation such as the dynamic convolution for accelerating CNN [218], 2) explicit knowledge for data augmentation by designing the failure surrogate model [55], or 3) metric to design conflicting loss between ML and physical predictions [176].

8. 2blueAbout when to use what form of knowledge. According to the analysis of the existing research in Table. 7, this paper suggests When dealing with explicit knowledge, the system behaviors can be described by mathematical equations or analytical models with clear input-output relations. If the physics signal variables involved in the equations are available, we can use them to customize NN layers or units based on analytical formulas or system physics characteristics. When explicit knowledge of the system behaviors is unavailable, but we have information about system physical structures or behavior model derivation knowledge, along with handling inference relations for the involved signals, we can construct a physics-informed structure. Embedded knowledge is then utilized to customize the data flow in the NN structure or employ custom-designed NNs as surrogate models for specific steps in the physics model derivation. In the case of tacit knowledge, where quantitative information about system behaviors is lacking, but there exist physical relations between system inputs and outputs or constraints on the system outputs, these relations can be employed to customize the ML objective function or regulate the output of hidden layers.

It is important to note that these forms of knowledge are not mutually exclusive, and they can often be combined or integrated within a PIML framework. The choice of which form to use, or whether to combine them, depends on the specific requirements and objectives of the problem at hand. A comprehensive approach to PIML may involve utilizing a combination of explicit, embedded, and tacit knowledge to capture the full range of system characteristics and optimize model performance. Ultimately, the selection of the appropriate form (s) of knowledge requires careful consideration of the problem domain, available expertise, data availability, and the desired level of interpretability and accuracy in the modeling process.

5. Challenges and future research directions: Toward PHM in the context of "small data and scarce physics knowledge"

Although PIML can bring numerous alternative solutions for diverse applications in PHM, as mentioned in previous sections, the development of PIML in PHM still comes with some particular limitations and challenges. Some of these limitations and challenges are presented in this section.

5.1. Limitations and challenges of PIML

The challenges related to sparse and noisy data, data availability, and incomplete physic models have been highlighted in other reviews [32, 18, 29]. In addition, there is a need for further research into the collection of more representative data, the selection of an appropriate benchmark model, and the determination of the weighting parameters or hyperparameters for the informed ML part. This paper argues that the fundamental problem underlying these challenges is how to convert various forms of knowledge into the type necessary for the ML framework, that is, to adjust knowledge to ML models rather than selecting ML models to fit physics knowledge. Consequently, one can highlight the challenges in two aspects:

1. **Building a physics aware ML framework**
   In the current research on the design of PIML, the inter-conversion between physics knowledge and ML, as well as the assessment metric for ML physical inconsistency, remains an under-explored and challenging topic. Therefore, there is a need to construct a physics-aware ML framework that can automatically incorporate physics knowledge into various parts of the ML pipeline based on the form of knowledge according to the physics-ML inter-conversion mechanism and the inconsistency evaluation results.

2. **Construction of knowledge basis**
   The construction of effective PIML frameworks requires a thorough comprehension of the physical characteristics of the system under investigation. This necessitates that the researcher possesses comprehensive knowledge and skills in both computer science and physics (e.g. mechanics). Thus, this leads to significant barriers for junior researchers without interdisciplinary experience in the implementation of PIML techniques. Furthermore, due to the fact that most knowledge can be ambiguous, mathematical expressions and physics modeling are not always well-developed for PHM case studies. Consequently, it is essential to construct a knowledge basis for PIML from ambiguous information.

5.2. Future directions

Although it is difficult to determine which direction will lead to transformative discoveries, the PHM community can make a substantial contribution to the future of PIML by taking into account the challenges outlined in the previous section in the following areas:

1. **Compiling fault mechanisms in ML**

   There is a lack of studies regarding how ML can better use integrated physics knowledge from multiple sources. Establishing the cognitive mechanisms for PIML when different forms of knowledge are embedded in different ML frameworks thus appears as a meaningful work.

2. **From the perspective of metric learning**

   Another track that appears worth investigation is the development of ML objective functions with more multidimensional metrics that can cover model complexity, physics consistency, computational cost, and result accuracy.

3. **Further building suitable benchmark problems**

   A set of well-defined benchmark problems can facilitate the evaluation and comparison of different algorithms and thus contribute to the development of research on this topic. The physics knowledge corresponding to public datasets is usually not available because of a deep-domain background requirement. This often leads to numerous obstacles in developing PIML methods because of the knowledge shortage. The development of open-source benchmark problems with shared encoded knowledge, such as the synthetic fatigue damage database presented in [219], or similar developments in other domains would be welcome.

4. **Combining flexible twin model to build life cycle tool**

   2blueThe modeling techniques and twinning enabling technologies in [192, 193] present an appealing opportunity to integrate with PIML. This combination has the potential to develop a robust life-cycle management tool that incorporates flexible fidelity models and maintains physics consistency. Additionally, it enables the inclusion of indeterminate quantitative assessment evaluations by building a twin framework.

5. **Adaptation to sparse run-to-failure data and sparse label data**

   Most of the research on PIML in PHM focuses on sparse observations or information bias in the dataset. However, unlabeled data and non-complete failure process data account for the vast majority. Therefore, the theoretical and applied research of PIML for sparse and noisy data is worth exploring. Development of new PHM paradigms such as physics-informed unsupervised learning, self-supervised learning, and semi-supervised learning is expected.

6. **PIML in “small data and scarce physics knowledge”**

   One of the ultimate development purposes of PIML in PHM is to be able to work under “small data and scarce physics knowledge” conditions, which will greatly extend the scope of potential PHM applications. Following the above-mentioned research directions, combining PIML with knowledge discovery and ML architecture search techniques has the potential for emerging breakthrough methods.

6. Conclusion

This paper presented bibliometric results and a state-of-the-art review of physics-informed machine learning for prognostics and health management. An overview of its basic paradigm from the innovative perspective of "How to inform" and "What to be informed", is summarized and provided. The existing approaches are grouped according to where and how the physics knowledge is embedded in the ML pipeline. The challenges of applying PIML to solve PHM problems and future research directions are also discussed. The main contribution of this paper is to provide a concise and comprehensive overview of PIML in PHM and to focus on the fundamental challenge of translating the appropriate form of knowledge for PIML utilization. It is hoped that this paper will encourage further works to expand the potential of PIML in the field of PHM.

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