Determination of Paris' law constants and crack length evolution via Extended and Unscented Kalman filter: An Application to Aircraft Fuselage Panels

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Abstract

Prediction of fatigue crack length in aircraft fuselage panels is one of the key issues for aircraft structural safety since it helps prevent catastrophic failures. Accurate estimation of crack length propagation is also meaningful for helping develop aircraft maintenance strategies. Paris' law is often used to capture the dynamics of fatigue crack propagation in metallic material. However, uncertainties are often present in the crack growth model, measured crack size and pressure differential in each flight and need to be accounted for accurate prediction. The aim of this paper is to estimate the two unknown Paris' law constants *m* and *C* as well as the crack length evolution by taking into account these uncertainties. Due to the nonlinear nature of the Paris' law, we propose here an on-line estimation algorithm based on two widespread nonlinear filtering techniques, Extended Kalman filter (EKF) and Unsented Kalman filter (UKF). The numerical experiments indicate that both EKF and UKF estimated the crack length well and accurately identified the unknown parameters. Although UKF is theoretical superior to EKF, in this Paris' law application EKF is comparable in accuracy to UKF and requires less computational expense.

Keywords: Crack growth, Paris' law constants, Extended Kalman filter, Unscented Kalman filter, Uncertainty

1. Introduction

Fatigue damage is one of the major failure modes of aircraft structures. Especially, repeated pressurization/depressurization cycles during take-off and landing cause many loading and unloading cycles which lead to fatigue crack in the fuselage panels. Prediction of fatigue crack propagation and estimation of remaining useful life (RUL) can be implemented to improve maintenance strategies [1,2]. This is particularly crucial for condition-based maintenance, which try to reduce unnecessary scheduled maintenance stops. The well-known Paris' law is widely used for simulating fatigue crack propagation in metallic materials. The two Paris' law constants {m, C} are treated as parameters to be estimated by the Kalman filtering framework. This makes sense as for an aircraft containing hundreds of fuselage panels, it is normal that the variability are present in the material property parameter among the panel population. Although the nominal of the Paris' law constants can be known since the material used to construct large commercial aircraft can be well characterized, fatigue is very dependent on minor microstructure irregularities which are difficult to control during the manufacturing process. This leads to various production batches of the material (Al-alloy) having somewhat different fatigue properties. In a large fleet of aircraft it thus cannot be assumed that all panels of all aircraft have the same fatigue properties as the nominal specification of the material manufacturer. Currently to account for this variability between different panels of an aircraft and between different aircraft, large safety factors are used when calculating fatigue life: a safety factor of 3 is typically assumed by the major manufacturers [3,4]. The corresponding variability is thus significant. Variability in $\{m, C\}$ present through the panel population in a fleet affects the crack propagation rate of each panel, resulting in different time-to-fail for each panel. This fact is important for the operation and management team of an airline to plan the optimal maintenance strategy. It is with this motivation that the Paris' law constants are treated as parameters that need to be identified for each individual panel. This is beneficial for crack growth prognosis and further for optimizing aircraft maintenance decision-making. In addition, we also consider the uncertainty present during crack propagation process, which is characterized by assuming the pressure differential is uncertain and varies in each flight cycle. The aim of this paper is to estimate the crack length evolution as well as the two unknown Paris' law constants {m, C} taking into account the above mentioned uncertainties by incorporating data from on-line monitoring system or non-destructive testing measurement [2,5].

The problem of state-parameter estimation is typically handled by resorting to filtering methods. The nonlinear nature of fatigue crack propagation leads to a nonlinear filtering problem. The two types of widely used filtering methods for nonlinear problem include the particle filter [6-8] and the variants of Kalman filter such as Extended Kalman filter (EKF) and the Unscented Kalman filter (UKF). The EKF and UKF are limited to handle the Gaussian noise while particle filter does not have this constrain. After formulating this non-linear filtering problem, we propose an on-line estimation/prediction algorithm based on the EKF and UKF for their strength in computational cost. This is particular important and results in cost saving when dealing with multi-component system where each component has its own dynamics.

The EKF is a commonly used algorithm for recursive nonlinear state-parameter identification due to its excellent filtering properties. A central and vital operation performed in EKF is the propagation of a random variable through the system dynamics [9]. In the EKF, the state dynamics model is expanded as a Taylor series around the prior mean of state variable. By ignoring the second and higher order terms, the state prediction propagates analytically through the nonlinear system equation whilst the state error covariance propagates through a separate first-order linearization of the nonlinear system [10]. Due to this linear approximation, EKF introduces errors from the second order in the true posterior mean and covariance of the transformed state variable, which may lead to sub-optimal results when dealing with significant nonlinearities. However, despite these approximations, from a practical application perspective, EKF algorithm remains a powerful tool in the nonlinear system state estimation domain and has been successfully used in various engineering state-parameter identification problems [11-14]. D'Alfonso et al.[15] used the EKF to estimate the position and orientation of a mobile robot, called

Khepera III, a battery-powered mobile robot with two independent driving wheels. 5 ultrasonic sensors for medium-range detection (from 25mm to 4m) were equipped in the robot. The data collected from these sensors were used to estimate the position and the orientation of the robot. Kim et al. [16] employed the EKF to estimate the hydrodynamic coefficients (specifically the 15 linear damping coefficients) of the AUV-SNUUV I, an autonomous underwater vehicle developed by Seoul National University. During the experiment, the input/output data were measured for SNUUV I in towing tank. The measured output signals were stored in onboard computer during experiment and these were transferred to the host computer through wireless LAN. The measured data were used to estimate the hydrodynamic coefficients. The results showed that after the initial transition period, all the 15 linear damping coefficients converge to some stable values. Bressel et al.[17] used the EKF to estimate the state health of proton exchange membrane fuel cell (PEMFC). The considered PEMFC is a commercially available five cells stack with an area of 100 cm² and with a nominal current of 60 A. This stack was operated under a constant load (60A) for 1500 h. The stack voltage was collected as the measurement data, from which the health state of the PEMFC was estimated. The experiments showed that the EKF offered good results in the health state estimation of the PEMFC.

The UKF has been proposed by Julier and Uhlmann in 1990s as a theoretically improved alternative to EKF for calculating the statistics of a random variable which undergoes a nonlinear transformation [10,18]. In UKF, a set of points (called sigma points) are chosen according to a specific selection algorithm so that the mean and covariance of these sigma points are equal to the state posterior mean and state error covariance. Each of these points propagates in turn through the nonlinear system equation to yield a set of transformed points of which the statistics are calculated as the prior estimates of the transformed state variable. Julier and Uhlmann demonstrated theoretically that compared with EKF, the state value estimated by UKF agreed with the true value up to the third order and errors were introduced in the fourth and higher order terms. Both EKF and UKF predicted the state error covariance up to the second order, but the absolute errors in the fourth and higher order terms for UKF were smaller than those of EKF [10].

The UKF has received great attention since it was presented and several studies compared it with EKF. VanDyke et al. [19] applied UKF to implement spacecraft attitude state-parameter estimation and compared the results with EKF. They argued that their UKF consistently outperform the EKF. Crassidis and Markley [20] also considered UKF for a spacecraft attitude estimation problem and claimed that for realistic conditions, especially when large initialization errors were present, UKF performed better than EKF. Qu and Hahn [21] used UKF for process monitoring and parameter estimation and argued that UKF outperformed EKF when severe nonlinearities exist and the measurement noise levels were high. However, although UKF has been proved to be a theoretical better approach to EKF, some recent research indicate that UKF often shows only a slight improvement and remains comparable in performance to EKF in some cases especially when the system is only moderately nonlinear. D'Alfonso et al. [15] employed both the EKF and UKF to estimate the position and orientation of a mobile robot. They ran their experiments 20 times to evaluate the performance of the EKF and the UKF, and found that the two filters performed comparably well in constructing the position of the robot and they attributed this to the fact that the nonlinearities in the model were not bad enough to highlight any substantial difference. Chowdhary and Jategaonkar [12] employed UKF and EKF to implement aerodynamics parameter estimation from flight data. Their results indicated that with a nonlinear model, no great difference between the numerical values of parameters was seen and UKF showed little improvement in time to convergence as compared to EKF. Wendel et al. [22,23] compared EKF and sigma point Kalman filter for nonlinear problem of tightly coupled GPS/INS integration and pointed out due to the fact that the higher-order transformation terms were negligible in the GPS/INS integration, these two methods offered an identical performance, which inferred that a modification of existing EKF-based navigation systems may not result in significant performance improvement. Qu and Hahn [21] applied UKF to a nonlinear process and drew similar conclusion: both UKF and EKF performed comparably well and the difference in the results achieved by these two estimators was minor when the system did not exhibit a strong extent of nonlinearity. Therefore, some researchers hold a conservative attitude for replacing EKF with UKF in practical application.

This paper is organized as follow. In section 2, we introduce both the crack growth model of the fuselage panel as well as the uncertainty model. In section 3, we formulate the estimation/prediction problem in a framework directly useable for EKF and UKF. Section 4 presents numerical results for both EKF and UKF on a short range commercial aircraft. Comparison in terms of efficiency and accuracy are drawn. Finally in section 5 conclusions and future work are presented.

2. Crack growth model

During the lifetime of an aircraft, loading and unloading cycles occur due to repeated pressurization/depressurization of the fuselage and can lead to fatigue cracks in the fuselage panels. Cracks or damages in this paper refer to existing flaws on the fuselage panel of an aircraft and are modeled as through-the-thickness center straight cracks in an infinite plate. This assumption is well verified if the crack size is small compared to the distance between fuselage stiffeners. For larger crack sizes the model can be adjusted by considering corrective terms in the calculations of the stress intensity factors to account for boundary conditions effect of stiffeners. Crack propagation can be modeled in myriad ways depending on different phenomena to which the critical crack site is subject [24-26]. Based on airframe fatigue tests on various military aircrafts, Molent et al. concluded that a simple crack growth model adequately represented a typical crack growth [27]. In this work, the celebrated Paris' law is selected to describe the crack growth behavior since it is commonly used for fatigue analysis due to its simplicity. The Paris' law is given by [28]:

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

where *a* is the half-crack size in meters, *N* is the number of load cycles. *da/dN* is the crack growth rate in meter/cycle. *C* and *m* are the Paris' law parameters which are associated with material properties. ΔK is the range of stress intensity factor in $MPa\sqrt{m}$, which is given in Eq. (2) as a function of the pressure differential (*p*), fuselage radius (*r*) and panel thickness (*t*).

$$\Delta K = \frac{pr}{t} \sqrt{\pi a} \tag{2}$$

Although the crack propagation is a continuous accumulation process, the length of crack size is measured every flight cycle in practice, which can be modeled by a discrete process. Furthermore, the EKF algorithm that we seek to apply usually needs to be implemented numerically and there might not be adequate computational power to integrate the system dynamics as necessary in a continuous-time EKF. Hence, system dynamics is discretized such that a discrete-time EKF can be used [29]. Euler method is employed to discretize Eq.(1) and the discrete Paris' law can be written in a recursive form at each flight cycle k as

$$a_{k} = a_{k-1} + C \left(\frac{p_{k-1}r}{t} \sqrt{\pi a_{k-1}} \right)^{m}$$

$$= g(a_{k-1}, p_{k-1})$$
(3)

Note that *m* and *C* are the parameters associated with material properties and remain constant once determined, while the pressure differential *p* can vary at every flight cycle. Then at each cycle, the pressure p_k is a random variable which is expressed as

$$p_{k} = \overline{p} + \Delta p_{k} \tag{4}$$

The disturbance Δp_k around the given average pressure \overline{p} is modeled as a centered normal distribution with variance σ^2 . Note that the corresponding variations in p_k are intended to model variations in cruise altitudes of the aircraft for different flights. It would also be possible to model different cruise altitudes by variations in \overline{p} directly but in this work we considered to model this through a perturbation term, given that the various possible cruise altitudes are relatively close to each other for a given aircraft.

Then Eq.(3) becomes

$$a_k = g(a_{k-1}, \overline{p} + \Delta p_{k-1}) \tag{5}$$

Since uncertainty on pressure is generally small, a Mean-Value First Order Second Moment (MVFOSM) approach [30] based on first order Taylor expression is considered in this paper. This gives:

$$a_{k} = g(a_{k-1}, \overline{p}) + \frac{\partial g(a_{k-1}, \overline{p})}{\partial p} \Delta p_{k-1}$$
(6)

where $(\partial g(a_{k-1}, \overline{p})/\partial p)$ is the first order partial derivative of g with respect to the variable p at the point (a_{k-1}, \overline{p}) and can be obtained analytically:

$$\frac{\partial g(a_{k-1},\overline{p})}{\partial p} = Cm(r/t)^m(\overline{p})^{m-1}(\pi a_k)^{m/2}$$

Taking $(\partial g(a_{k-1}, \overline{p})/\partial p) \Delta p_{k-1}$ as the addictive process noise and considering that \overline{p} is a given constant, Eq.(6) could be written as:

$$a_{k} = f(a_{k-1}) + w_{k-1}$$
where
$$f(a_{k-1}) = g(a_{k-1}, \overline{a}) \text{ and}$$
(7)

$$w_{k-1} = \frac{\partial g(a_{k-1}, \overline{p})}{\partial p} \Delta p_{k-1}$$
(8)

According to Eq.(8) the additive process noise w_k at each cycle follows a normal distribution with mean zero and variance Q_k , given in Eq.(9).

$$Q_{k} = \left(\frac{\partial g(a_{k-1}, \overline{p})}{\partial p} \sigma_{p}\right)^{2}$$

$$= \left(Cm(r/t)^{m}(\overline{p})^{m-1}(\pi a_{k})^{m/2} \sigma_{p}\right)^{2}$$
(9)

Since the crack size is measured by sensors, the measured crack size always contains noise due to the measurement environment and sensor limitations. The measurement data is modeled as

 $z_k = h(a_k) + v_k$

in which *h* is the measurement function and v_k is the measurement noise such that $v_k \sim N(0,R)$. In this paper, the measurement function *h* is identity. Eq.(7) and Eq.(10) are called the system equation and measurement equation respectively. Eq.(10) is used to simulate the actual measurement data since at this stage, it is difficult to get experimental data on aircraft fuselage panel to be used directly on our approach. The application of SHM systems in commercial aircraft is still at the research stage and its widespread application to airlines has a long way to go. Tests have been done during the last decades by airlines as well as research centers. For example, very recently, seven of the Boeing Co.'s 737 narrow body aircraft operated by Delta Air Lines have been outfitted with Comparative Vacuum Monitoring sensors for crack detection in a program that aims to obtain approval for SHM as an alternative inspection method by 2016. Part of a broader SHM initiative at the Airworthiness Assurance Nondestructive Inspection Validation Center operated by Sandia National Labs for the US Federal Aviation Admin (FAA). In addition, the major aircraft OEMs as well as operators, regulators and technology suppliers have been striving for years to standardize SHM integration and certification requirements and to mature system for widespread use. Note that in Eq.(10) a normality assumption is made on the crack size measurements noise. While we do not have experimental data for the considered application problem to back this up, normal measurement noise on the crack size is

(10)

typically considered when dealing with fatigue crack growth [31-33]. Furthermore we will show in the results section that the proposed approach is relatively robust to the actual measurement distribution.

3. State and parameter estimation

Estimation of parameters by a filtering approach can be classified into two categories, joint filtering [19] and dual filtering [34-36]. Both of them use a similar filter to estimate state and parameters simultaneously. Joint filtering, as the simpler of the two, has been chosen here for this first attempt of joint estimation of the Paris' law material properties and crack size by EFK and UKF. Joint filtering defines the parameter vector of interest as an additional state variable and artificially appends it onto the true state vector. The appended portion of augmented state vector does not change beyond the effects of process noise during the time-update process while the augmented error covariance matrix is propagated as a whole (i.e. the parameters inherently do not depend on time evolution and keep constant).

3.1 Estimation framework

In the aforementioned crack growth model, m and C are the unknown parameters that need to be estimated. Therefore, a twodimensional parameter vector is defined as

 $\Theta = [m, C]^{T}$ (11)
Appending Θ to the state variable, that is crack length *a*, the augmented state vector is defined as $x_{aug} = [a \ m \ C]^{T}$ (12)

Using subscript "*aug*" to denote all the augmented variables, the extended system equation is represented as Eq.(13) and expanded in the form of matrix for the sake of clarity, given by Eq.(14).

$$\begin{aligned} x_{aug,k} &= f_{aug}(x_{aug,k-1}) + w_{aug,k-1} \\ \begin{bmatrix} a_k \\ m_k \end{bmatrix} &= \begin{bmatrix} f(a_{k-1}) \\ m_{k-1} \\ C_{k-1} \end{bmatrix} + \begin{bmatrix} w_{k-1} \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$
(13)

The augmented process noise covariance matrix, denoted as $Q_{aug,k}$ is written in Eq.(15), in which Q_k is the variance of process noise on crack length given in Eq.(9).

$$Q_{aug,k} = \begin{bmatrix} Q_k & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = diag(Q_k, 0, 0)$$
(15)

The measurement equation should also be extended, which is represented as Eq.(16) and the matrix form is given in (17).

$$z_{aug,k} = h_{aug}(x_{aug,k}) + v_{aug,k}$$

$$(16)$$

$$\begin{bmatrix} z_{a,k} \\ z_{m,k} \\ z_{C,k} \end{bmatrix} = \begin{bmatrix} a_k \\ m_k \\ C_k \end{bmatrix} + \begin{bmatrix} v_{a,k} \\ v_{m,k} \\ v_{C,k} \end{bmatrix}$$
(17)

where $v_{a,k}$, $v_{mv,k}$ and $v_{C,k}$ represent respectively the uncorrelated measurement noise of each state variable with a zero mean and a variance of R_a , R_m , R_C . The augmented measurement noise covariance matrix R_{aug} could be written as $R_{aug} = diag(R_a, R_m, R_C)$ (18)

3.2 The Extended Kalman filter algorithm

The details of the EKF algorithm [37,38] applied to the crack propagation problem are provided in this section. We use the symbol " \wedge " to represent an estimate and subscript "k" to denote the time step. Symbols "–" and "+" in the upper right corner are used to indicate a prior estimate and a posterior estimate respectively. For example, $\hat{x}_{aug,k}^-$ represents a prior estimate of the augmented state vector at time step k while $\hat{x}_{aug,k}^+$ denotes the posterior estimate at the same time. Similar, P_k^- is the a prior estimate for state error covariance matrix at time step k while P_k^+ is the posterior one. For the augmented system of Eq.(13) and Eq.(16), the EKF consists of initialization, extrapolation and update.

Initialization:

Four variables, process noise covariance matrix Q_{aug} , measurement noise covariance matrix R_{aug} , estimated initial state value $x_{aug,0}$ and initial state error covariance matrix P_0 should be initialized. Q_{aug} is a 3-by-3 matrix including only one nonzero element Q_k , which is given in Eq.(9). R_{aug} is a 3-by-3 diagonal matrix and each diagonal element (R_a , R_m , R_c) needs to be determined separately. R_a is the variance on measured crack size and can be obtained from the sensor specifications, while R_m and R_c are generally defined as a percentage of the order of magnitude of m and C. In this paper, R_{aug} is constant and time invariant. $x_{aug,0}$ is a 3-by-1 vector contains 3 elements, \hat{a}_0 , \hat{m}_0 and \hat{C}_0 , which are the initial estimate for crack length a, and parameters $\{m, C\}$, respectively. In this paper, \hat{a}_0 , \hat{m}_0 and \hat{C}_0 are generated randomly from uniform distribution. P_0 represents the confidence in the initial estimate for state. In the absence of any a prior knowledge of the initial state values it is common to assume high value for the P_0 [12]. In contrast, if one is confident to the initial estimate for state, P_0 is generally small.

Extrapolation:

According to EKF recurrence formulas, the system propagates as follow.

 $\hat{x}_{aug,k}^{-} = f_{aug}(\hat{x}_{aug,k-1}^{+})$

Rewriting Eq.(19) in matrix form for sake of clarity, the above equation is equivalent to:

$$\begin{bmatrix} \hat{a}_{k}^{-} \\ \hat{m}_{k}^{-} \\ \hat{C}_{k}^{-} \end{bmatrix} = \begin{bmatrix} f(\hat{a}_{k-1}^{+}) \\ \hat{m}_{k-1}^{+} \\ \hat{C}_{k-1}^{+} \end{bmatrix}$$
(20)

The error covariance *P* propagates as follow:

$$P_{k}^{-} = \Phi_{k-1} P_{k-1}^{+} \Phi_{k-1}^{-T} + Q_{aug,k-1}$$
(21)

where Φ is the Jacobian matrix of the augmented system equation f_{aug} at point $\hat{x}_{aue,k-1}^- = [\hat{a}_{k-1}^-, \hat{m}_{k-1}^-, \hat{C}_{k-1}^-]^T$. It is computed as follows:

$$\Phi_{k-1} = \frac{\partial f_{aug}(\hat{x}_{aug,k-1})}{\partial x_{aug}}$$

$$= \begin{bmatrix} 1 + C \frac{m}{2} \pi^{\frac{m}{2}} (\frac{pr}{t})^m (a)^{\frac{m}{2}-1} & C(\frac{pr}{t} \sqrt{\pi a})^m \ln(\frac{pr}{t} \sqrt{\pi a}) & (\frac{pr}{t} \sqrt{\pi a})^m \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{a=\hat{a}_{k-1}^-, m=\hat{m}_{k-1}^-, C=\hat{C}_{k-1}^-}$$
(22)

Update:

The Kalman gain K_k is computed from Eq.(23). In this case, K_k is a 3-by-3 matrix. $K_{k} = P_{k}^{-}H_{k}[H_{k}P_{k}^{-}H_{k}^{T} + R_{aue,k}]^{-1}$

where *H* is the Jacobin matrix of the augmented measurement equation h_{aug} at point $\hat{x}_{aug,k}^- = [\hat{a}_k^-, \hat{m}_k^-, \hat{C}_k^-]^T$ given as:

$$H_{k} = \frac{\partial h_{aug} \left(\hat{x}_{aug,k}^{-} \right)}{\partial x_{aug}} \tag{24}$$

=diag(1,1,1)

The estimated measurement can be computed as

$$\hat{z}_{aug,k} = h_{aug}(\hat{x}_{aug,k}) \tag{25}$$

Expanding the above equation in matrix form, it is equivalent to:

$$\begin{bmatrix} \hat{z}_{a,k} \\ \hat{z}_{m,k} \\ \hat{z}_{C,k} \end{bmatrix} = \begin{bmatrix} \hat{a}_k^- \\ \hat{m}_k^- \\ \hat{C}_k^- \end{bmatrix}$$
(26)

The posterior estimate of state is obtained from Eq.(27) and it is expanded in matrix form for clarity, as given in Eq.(28)

$$\hat{x}_{aug,k}^{+} = \hat{x}_{aug,k}^{-} + K_{k}(z_{aug,k} - \hat{z}_{aug,k})$$

$$\begin{bmatrix}
\hat{a}_{k}^{+} \\
\hat{m}_{k}^{+} \\
\hat{c}_{k}^{+}
\end{bmatrix} = \begin{bmatrix}
\hat{a}_{k}^{-} \\
\hat{m}_{k}^{-} \\
\hat{c}_{k}^{-}
\end{bmatrix} + K_{k}(\begin{bmatrix}
z_{a,k} \\
z_{m,k} \\
z_{C,k}
\end{bmatrix} - \begin{bmatrix}
\hat{z}_{a,k} \\
\hat{z}_{m,k} \\
\hat{z}_{C,k}
\end{bmatrix})$$
(27)
$$(28)$$

The error covariance matrix is updated as follow:

 $P_k^+ = [I - K_k H_k] P_k^-$ (29) To summarize, the different steps of the EKF algorithm implementation are given in Table 1. Table 1 Pseudo code of EKF

1. Initialize $\hat{x}_{aug,0}$, P_0 For each time step: k = 1, 2...end2. Compute the system equation Jacobian matrix: $\Phi_{k-1} = \frac{\partial f_{aug}(\hat{x}_{aug,k-1}^{-})}{\partial x_{aug}}$ 3. Perform the extrapolation process of the state and error covariance as follows: $\hat{x}_{aug,k}^{-} = f_{aug}(\hat{x}_{aug,k-1}^{+})$

$$P_{k}^{-} = \Phi_{k-1} P_{k-1}^{+} \Phi_{k-1}^{T} + Q_{aug,k-1}$$

4. Compute the measurement equation Jacobian matrix:

(19)

(23)

(07)

 $H_{k} = \frac{\partial h_{aug}(\hat{x}_{aug,k}^{-})}{\partial x_{aug}}, \text{ here } H_{k} = diag(1,1,1)$ 5. Calculate the Kalman gain $K_{k} = P_{k}^{-}H_{k}[H_{k}P_{k}^{-}H_{k}^{-T} + R_{aug,k}]^{-1}$ 6. Perform the measurement update of the state and error covariance as follows: $\hat{x}_{aug,k}^{+} = \hat{x}_{aug,k}^{-} + K_{k}(z_{aug,k} - \hat{z}_{aug,k})$ $P_{k}^{+} = [I - K_{k}H_{k}]P_{k}^{-}$

3.3 The Unscented Kalman filter algorithm

The UKF is based on the idea that it is easier to approximate a probability distribution than to approximate an arbitrary nonlinear transformation [39]. The algorithm is based on propagating carefully selected finite set of points, called sigma points, through the system nonlinear dynamics, and then approximating the first two moments of the distribution (mean and covariance) through a suitable method; such as weighted sample mean and covariance calculations [9,39]. Studies on the theoretical framework of the UKF algorithm can be found in [10,18]. For the augmented system in Eq.(13) and Eq.(16), the procedure of propagating the system state from time step k-1 to time step k through UKF is explained below.

(1). The filter is initialized with the estimated state mean $\hat{x}_{aug,0}$ and state error covariance matrix P_0 .

(2).2*n*+1 points, called sigma points (χ_i , *i* = 0,1...2*n*), are calculated based on the state posterior mean and error covariance matrix, where *n* is the dimension of the state vector, here *n*=3. Supposing at time step *k*-1, the posterior mean and error covariance are $\hat{x}^+_{aug,k-1}$ and P^+_{k-1} . The sigma points are calculated as:

$$\chi_{0,k-1} = \hat{x}_{aug,k-1}^{+}$$

$$\chi_{i,k-1} = \sigma_{i,k-1} + \hat{x}_{aug,k-1}^{+} \qquad i = 1,2...2n$$
(30)

where

$$\sigma_{i,k-1} = (\sqrt{(n+\kappa)P_{k-1}^+})_i \quad \text{for } i = 1, 2...n$$

$$\sigma_{i+n,k-1} = -(\sqrt{(n+\kappa)P_{k-1}^+})_i \text{ for } i = 1, 2...n$$
(31)

Note that index $(\bullet)_i$ denotes the *i*-th column of the matrix (\bullet) . The matrix's square root can be calculated by using a lower triangular Cholesky factorization method that prevents the negative covariance matrix [40]. κ is a scaling parameter that we will detail later. It provides an extra degree of freedom to "fine tune" the higher order moments of the approximation, and can be used to reduce the overall prediction error [18]. The process of generating σ and the corresponding sigma points (in our case, $\sigma_i, i = 1, ..., 6$ and $\chi_i, i = 0, 1..., 6$) at time step *k*-lfor the crack propagation problem considered are detailed below. Let us

denote the matrix $\sqrt{(n+\kappa)P_{k-1}^+}$ as below:

$$\sqrt{(n+\kappa)P_{k-1}^{+}} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$
(32)

Then σ_i is obtained as follow:

$$\begin{bmatrix} \sigma_{1} & \sigma_{2} & \sigma_{3} & \sigma_{4} & \sigma_{5} & \sigma_{6} \\ \hline P_{11} & P_{12} & P_{13} & -P_{11} & -P_{12} & -P_{13} \\ P_{21} & P_{22} & P_{23} & -P_{21} & -P_{22} & -P_{23} \\ \hline P_{31} & P_{32} & P_{33} & -P_{31} & -P_{32} & -P_{33} \end{bmatrix}$$
(33)

Each sigma point $\chi_{i,k-1}$ is a 3-by-1 vector and the 7 sigma points constitute a 3-by-7 matrix which is expressed below:

$$\chi_{k-1} = \begin{bmatrix} \chi_{0,k-1} & \chi_{1,k-1} & \chi_{2,k-1} & \chi_{3,k-1} & \chi_{4,k-1} & \chi_{5,k-1} & \chi_{6,k-1} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \hat{m}_{k-1}^{+} & \vdots & \vdots & \vdots \\ \hat{m}_{k-1}^{+} & \vdots & \vdots & \vdots \\ \hat{m}_{k-1}^{+} & \vdots & \vdots & \vdots \\ \hat{C}_{k-1}^{+} & \vdots & \vdots & \vdots \\ \hat{C}_{k-1}^{+} & \vdots \\ \hat{C$$

These sigma points assure that

$$\sum_{i=1}^{n} W_i \chi_{i,k-1} = \hat{x}^+_{aug,k-1}$$

$$\sum_{i=1}^{n} W_i (\chi_{i,k-1} - \hat{x}^+_{aug,k-1}) (\chi_{i,k-1} - \hat{x}^+_{aug,k-1})^T = P^+_{k-1}$$
(35)

in which W is the weight of each point that can be calculated as:

$$W_0 = \frac{\kappa}{n+\kappa}$$

$$W_i = \frac{1}{2(n+\kappa)} \qquad i = 1, 2, \dots 2n$$
(36)

(3). Each sigma point propagates from k-1 to k through the nonlinear system equation:

 $\chi_{i,k} = f_{aug}(\chi_{i,k-1}) \tag{37}$

(4). The statistics of the propagated sigma points are calculated as the a prior mean and error covariance of the state at time step k: 2n

$$\hat{x}_{aug,k}^{-} = \sum_{i=0}^{2n} W_i \chi_{i,k}$$

$$P_k^{-} = \sum_{i=0}^{2n} W_i (\chi_{i,k} - \hat{x}_{aug,k}^{-}) (\chi_{i,k} - \hat{x}_{aug,k}^{-})^T$$
(38)

(5). Each sigma point $\chi_{i,k}$ transforms through the measurement equation to generate an estimated measurement $\gamma_{i,k}$.

$$\gamma_{i,k} = h_{aug}(\chi_{i,k})$$
(39)
(6).The mean and covariance of the estimated measurement are calculated based on the statistics of the transformed points:

$$\hat{z}_{aug,k} = \sum_{i=0}^{2n} W_i \gamma_{i,k}$$

$$P_{\hat{z}\hat{z}} = \sum_{i=0}^{2n} W_i (\gamma_{i,k} - \hat{z}_{aug,k}) (\gamma_{i,k} - \hat{z}_{aug,k})^T$$
(40)

(7). The cross-correlation covariance of state-measurement can be obtained as:

$$P_{x\hat{z}} = \sum_{i=0}^{2n} W_i (\chi_{i,k} - \hat{x}_{aug,k}) (\gamma_{i,k} - \hat{z}_{aug,k})^T$$
(41)

(8). The Kalman gain matrix is approximated from the cross-correlation covariance and the estimated measurement covariance as:

$$K_k = P_{x\hat{z}} P_{\hat{z}\hat{z}}^{-1} \tag{42}$$

(9). The posterior estimates of state mean and error covariance are:

$$\hat{x}_{aug,k}^{+} = \hat{x}_{aug,k}^{-} + K_{k}(z_{aug,k} - \hat{z}_{aug,k}) P_{k}^{+} = P_{k}^{-} + K_{k}P_{zz}^{-}K_{k}^{T}$$
(43)

The different steps of the UKF algorithm implementation are summarized in Table 2. Table 2 Pseudo code of UKF

1. Initialize $\hat{x}_{aug,0}$, P_0

For each time step: k = 1, 2...end

2. Generate the sigma point matrix χ_{k-1} , see Eq.(34)

3. Compute weight of each point as:

$$W_0 = \frac{\kappa}{n+\kappa}$$
$$W_i = \frac{1}{2(n+\kappa)} \qquad i = 1, 2, \dots 2n$$

4. Use the augmented system equation f_{aug} to transform each sigma point $\chi_{i,k-1}$ into $\chi_{i,k}$:

 $\chi_{i,k} = f_{aug}(\chi_{i,k-1})$

5. Predict the state mean and error covariance as:

$$\hat{x}_{aug,k}^{-} = \sum_{i=0}^{2n} W_i \chi_{i,k}$$

$$P_k^{-} = \sum_{i=0}^{2n} W_i (\chi_{i,k} - \hat{x}_{aug,k}^{-}) (\chi_{i,k} - \hat{x}_{aug,k}^{-})^T$$

6. Use the augmented measurement equation to transform each sigma point $\chi_{i,k-1}$ into $\gamma_{i,k}$:

$$\gamma_{i,k} = h_{aug}(\chi_{i,k})$$

7. Compute the mean and covariance of predicted measurement as:

$$\hat{z}_{aug,k} = \sum_{i=0}^{2n} W_i \gamma_{i,k}$$

$$P_{\hat{z}\hat{z}} = \sum_{i=0}^{2n} W_i (\gamma_{i,k} - \hat{z}_{aug,k}) (\gamma_{i,k} - \hat{z}_{aug,k})^T$$

8. Compute cross-correlation covariance of state-measurement

$$P_{x\hat{z}} = \sum_{i=0}^{m} W_i (\chi_{i,k} - \hat{x}_{aug,k}^-) (\gamma_{i,k} - \hat{z}_{aug,k})^T$$

9. Compute Kalman gain

 $K_{k} = P_{xz} P_{zz}^{-1}$ 10. Update the state mean and error covariance $\hat{x}_{aug,k}^{+} = \hat{x}_{aug,k}^{-} + K_{k} (z_{aug,k} - \hat{z}_{aug,k})$ $P_{k}^{+} = P_{k}^{-} + K_{k} P_{zz}^{-} K_{k}^{T}$

Different aspects that need to be considered during the implementation of the above procedures are summarized below:

(1). The scaling parameter κ in Eq.(31) affects the scaling of the fourth and higher order moments of the distribution of σ_i . It is often given by the following equation $\kappa = \alpha^2 (n+\lambda) - n$ where *n* is the dimension of the state vector. α controls the spread extent of the sigma points around the mean and is usually set to a small positive value (e.g.,1e-3). λ is the secondary scaling parameter usually set to be 0. The value of κ is crucial for the UKF estimator. An inappropriate choice of κ will cause the estimator divergence [9] [12].

(2).In the algorithm of UKF, the generation of sigma points depends on the calculation of the square root of the covariance matrix. Since the orthogonal or symmetric matrix square roots are numerically sensitive and computationally expensive to find, in practical, more efficient and stable methods such as the Cholesky decomposition are generally recommended to calculate the matrix square root [18].

(3).In general UKF, the process noise should be incorporated into the state vector, which makes the dimension of state vector augment from *n* to n+q, where *q* is the dimension of process noise vector. However, the process noise is assumed to be zeromean or additive in practice, which does not require augmenting the state vector with the noise variables, thus decreasing again the number of points required to be propagated through the nonlinear system from 2(n+q) to 2n[19].

4. Numerical study

4.1 EKF

In the present and the following sections we investigate the application of the EKF and UKF algorithms to the estimation of the parameters m and C and true crack size a based on noisy measurements of the crack size in an aircraft fuselage panel. The values in Table 3 are used for the numerical simulations. The values defining the aircraft geometry are characteristic of a short range commercial aircraft (e.g. A320, B737). The total number of simulation steps is set to be 60000 cycles based on the typical lifetime of such short-range commercial aircrafts. The measurement noise covariance matrix is composed of three items. The first one relates to the crack size and depends on the sensors capabilities. The second and third ones relate to the material properties and can stem from manufacturer's specification. This allows taking into account the manufacturer's estimation of the variability in the material properties.

The parameter vector Θ , composed of the two material property parameters $\{m, C\}$, has been defined in Eq.(11). At each step of the EKF procedure, the estimated parameters are random variables. Accordingly, 50 repetition of the EKF estimation process have been done to characterize this randomness, i.e., at each flight cycle *k* (*k*=1,2,...60000), 50 estimates (samples), denoted by $\hat{\Theta}_{k}$ (*j*=1,2...50), are obtained by the EKF estimator.

	Table 3	Parameters	for	numerical	study
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	ruere e r uru	neters for namerical stady		
Parameter	Denotation	Туре	Value	Unit
Fuselage radius	r	Derministic	1.95	m
Panel thickness	t	Derministic	2×10^{-3}	m
Pressure differential	р	Normally distributed	N(0.06,0.003)	MPa
True initial crack length	a_0	Derministic	2×10^{-4}	m
True Paris' law parameter	т	Derministic	3.8	-
True Paris' law parameter	С	Derministic	1.5×10^{-10}	-
Estimated initial crack length	\hat{a}_0	Uniformly distributed	$U(1.5 \times 10^{-4}, 2.5 \times 10^{-4})$	m
Estimated initial Paris' law parameter	\hat{m}_0	Uniformly distributed	<i>U</i> (2.85, 4.75)	-
Estimated initial Paris' law parameter	$\hat{C}_{_0}$	Uniformly distributed	$U(1.125 \times 10^{-10}, 1.875 \times 10^{-10})$	-
Initial error covariance matrix	P_0	Derministic	$diag(1 \times 10^{-4}, 1, 1 \times 10^{-10})$	-
Measurement noise covariance	Ra	Derministic	$(10\% \times a_0)^2$	-

Rm	Derministic	$(35\% \times m)^2$	-
RC	Derministic	$(35\% \times C)^2$	-

To characterize the convergence behavior of the EKF estimation over these 50 samples we computed 4 indicators, $\hat{\Theta}_k$, Θ_{error} , MSE_k and MSE_k , that will be explained in the following. $\hat{\Theta}_k$ is the average value of these 50 samples at the *k*-th flight cycle, as shown in Eq.(44), in which n_s is the number of repetitions of the simulations, here, n_s =50. Θ_{error} is the absolute relative error on $\hat{\Theta}_k$, calculated through Eq.(45), in which Θ denotes the "true" value of parameter.

1 1

$$\hat{\Theta}_{k} = \frac{1}{n_{s}} \sum_{j=1}^{s} \hat{\Theta}_{k,j}$$

$$\Theta_{error} = \frac{\left|\hat{\Theta}_{k} - \Theta\right|}{\Theta} \times 100\%$$
(44)
(45)

To measure the spread of the EKF estimation, the estimated variance at the *k*-th flight cycle in presence of the true values of the parameters, MSE_k , is computed through Eq.(46). Moreover, the estimated variance at *k*-th flight cycle in absence of the true values of the parameters, MSE_k , is also given by Eq.(47). This indicator can be useful for determining the confidence interval when the true values are unknown, which is typically the case in prognostics of residual life.

$$MSE_{k} = \frac{1}{n_{s} - 1} \sum_{j=1}^{n_{s}} (\hat{\Theta}_{k,j} - \Theta)^{2}$$
(46)

$$M\hat{S}E_{k} = \frac{1}{n_{s} - 1} \sum_{j=1}^{n_{s}} (\hat{\Theta}_{k,j} - \hat{\Theta}_{k})^{2}$$
(47)

Let us remind the difference between MSE and MSE given the above equations. MSE is used to indicate how far on average the collection of estimates are from the true values of parameters being estimated, which in practice, are generally unknown while MSE indicates how far, on average, the collection of estimates are from the average value of estimates. Table 4 listed the four indicators of the EKF estimator for *m* and *C* after 100, 1000, 10000, 30000 and 60000 cycles.

- K error								
No. of cycles	Para.	$\hat{\Theta}_k$	Θ_{error} (%)	MŜE	MSE			
100 avalas	m	3.80627	1.70E-01	1.220600E-02	1.224600E-02			
100 cycles	С	1.49429e-10	3.80E-01	2.155435E-23	2.188726E-23			
1000 avalas	m	3.79330	1.80E-01	1.762000E-03	1.808000E-03			
1000 cycles	С	1.49615e-10	2.60E-01	3.244177E-24	3.395274E-24			
10000 1	m	3.80116	3.10E-02	2.333102E-04	2.346924E-04			
10000 cycles	С	1.49998e-10	1.10E-03	2.949449E-25	2.949477E-25			
20000 avalas	m	3.79916	2.20E-02	5.354348E-05	5.426088E-05			
50000 cycles	С	1.49984e-10	1.10E-02	1.183051E-25	1.185798E-25			
60000 cycles	m	3.79968	8.40E-03	2.512509E-05	2.522799E-05			
	С	1.50003e-10	1.80E-03	5.214537E-26	5.215299E-26			

Table 4 $\hat{\Theta}_k / \Theta_{error} / M\hat{S}E / MSE$ of *m* and *C* over 50 simulations by EKF

The comparisons between the true value of parameter $\{m, C\}$ and the estimated value of parameter $\{\hat{m}_k \ \hat{C}_k\}$ at every cycle k are respectively presented in Fig.1 and Fig.2. Note that each of the figures contains two subfigures, the left one (marked as (a)) is the original figure to show the overall view while the right one (marked as (b)) has reduced the range of y-axis to show more clearly the convergence process towards the true parameter values. A similar manner will be employed in the following figures throughout this paper. It can be observed that both the two parameters are accurately identified and they rapidly converge to the true value. While the initial values of \hat{m} and \hat{C} are uniformly distributed with a range of 50% around the true values (see Table 3), on average, the error of the EKF estimate for *m* has already decreased to 0.17% after 100 cycles and that of *C* decreased to 0.38% after these same 100 cycles. These errors fluctuate to a small extent but remain very low through the remainder of the cycles (up to 60000 cycles): less than 0.32 % for *m* and less than 0.53% for *C*. 90% confidence interval (C.I.) on the estimations of \hat{m}_k / \hat{C}_k at each cycle for one run of the EKF algorithm have also been given in Fig.Fig.1 and Fig.2. Based on this C.I. we can note that the spread in the convergence behavior is also small (The spread of the C.I. is 9.73% for *m* and 10.42% for *C* after 100 cycles).

Fig.3 (a) illustrated the crack length evaluation estimated by EKF based on one simulation out of the 50 repetitions. Each point represents a noisy measurement of the crack length used in the EKF algorithm. Note that only one point every 100 cycles is represented in order not to overload the figure. For sake of observing more clearly the difference between the true crack size and the EKF estimation, the crack length evolution during the first 10000 cycles is also given in Fig.3 (b). It can be seen that the estimated crack length (dashed line) fits very well the true one (solid line) even with polluted measurements (solid points). For example, the error of EKF estimate for *a* attenuates to 1.27% after 100 cycles. Through the reminder of lifetime, the error fluctuates to a small degree but remains very low, less than 1.36% and overall declines progressively. The numerical experiments results indicate that EKF shows very good performance for estimating the crack length as well as the Paris' law constants *m* and *C*.



Fig.3 Crack length evolution by EKF based on one simulation, measurement points are plotted every 100 cycles

4.2 UKF

For numerical experiments of UKF, the same parameters of Table 3 are used. Similar to EKF, 50 simulations have been implemented and the 4 indicators, $\hat{\Theta}_k$, Θ_{error} , *MSE* and *MSE* have been computed. Table 5 gives these 4 indicators of UKF estimator for *m* and *C* after 100, 10000, 10000, 30000 and 60000 cycles.

Table 5 $\hat{\Theta}_k / \hat{\Theta}_{error} / MSE / MSE$ of <i>m</i> and <i>C</i> over 50 simulations by UKF									
No. of cycles	Para.	$\hat{\Theta}_k$	Θ_{error} (%)	MŜE	MSE				
100 cycles	m	3.817085	0.4800	2.183600E-02	2.213400E-02				
	С	1.497232e-10	0.1800	2.761814E-23	2.769628E-23				
1000 avalas	m	3.808947	0.2300	2.167000E-03	2.249000E-03				
1000 cycles	С	1.501541e-10	0.1000	2.953025E-24	2.977271E-24				
10000 cycles	m	3.800338	0.0089	1.940369E-04	1.941535E-04				
	С	1.501142e-10	0.0760	2.751670E-25	2.884843E-25				

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200001	m	3.799208	0.0210	8.212850E-05	8.276845E-05
50000 cycles	С	1.500206e-10	0.0130	1.181696E-25	1.186023E-25
<pre>(00001 ¹)</pre>	m	3.799486	0.0130	3.539677E-05	3.566595E-05
60000 cycles	С	1.500006e-10	0.0004	6.199578E-26	6.199615E-26

The comparisons between {m, \hat{m}_k } and {C, \hat{C}_k } at each cycle k given by UKF is respectively presented in Fig.4 and Fig.5. The results and conclusions are similar to the one found in EKF with rapid convergence of the parameters to their true values. For example, the error of UKF estimate decreases to 0.48% for m and 0.18% for C after 100 cycles. Through the remainder of cycles (from 100 cycles up to the aircraft's end of life), the errors oscillate slightly but keep very low: maximum 0.56% for m and 0.38% for C. Fig.4 and Fig.5 also gives 90% confidence interval (C.I.) on the estimations of { \hat{m}_k , \hat{C}_k } at each cycle for one run of UKF algorithm. Based on this C.I. we can note that the spread in the convergence behavior is also small (The spread of the C.I. is 12.98% for m and 11.77% for C after 100 cycles).

Crack length evolution based on one UKF simulation is illustrated in Fig.6 (a) and the first 10000 cycles is presented in Fig.6 (b). The estimated crack length (dashed line) showed again excellent agreement with the true one (solid one). For instance, the error declines to 0.89% after 100 cycles and keeps less than 0.89% until the end of life. The numerical experiments results indicate that UKF shows very good performance with the system under consideration.



Fig.6 Crack length evolution by UKF based on one simulation,

4.3 Comparison of EKF and UKF

In order to compare the performance of EKF against UKF, simulation with these two algorithms is conducted simultaneously 50 times under the same initial conditions and the same 4 indicators, $\hat{\Theta}_k$, Θ_{error} , MSE and MSE are calculated. Table 6 compared these two methods in terms of the 4 indicators for m and C after several flight cycles. The difference of the estimators $\hat{\Theta}_k$ given by EKF and that given by UKF are present in the 5th column in Table 6. It can be observed that the results of these two approaches are very close, which indicates that EKF results have a comparable accuracy to UKF. $/M\hat{S}F/MSE$ m and C over 50 simulations in minor nonlinear case Table 6 Comparison for $\hat{\Theta}$ / Θ

Table	o Compa		Θ_k / Θ_{erro}	or / MISE	MSE m and C OV	er 50	siniurati		ioi nonnneai case	
cucles	Dara	Filter	â		Difference	0	(0/)	MÂE	MSE	Ĩ

No. of cycles	Para.	Filter	$\hat{\Theta}_k$	Difference	Θ_{error} (%)	MŜE	MSE
100 cycles		EKF	3.80820994	1.00 - 08	2.20E-01	2.060300E-02	2.067200E-02
	III	UKF	3.80820998	1.096-08	2.20E-01	2.019100E-02	2.025800E-02
	C	EKF	1.50565698e-10	1 242 09	3.80E-01	2.438032E-23	2.470687E-23
	C	UKF	1.50565700e-10	1.54e-08	3.80E-01	2.389270E-23	2.421273E-23
	m	EKF	3.80590371	1.002.07	1.60E-01	1.974000E-03	2.010000E-03
1000 avalas	111	UKF	3.80590413	1.096-07	1.60E-01	1.935000E-03	1.970000E-03
1000 cycles	C	EKF	1.50147458e-10	1 102 07	9.80E-02	3.086431E-24	3.108620E-24
	C	UKF	1.50147474e-10	1.10e-07	9.80E-02	3.024703E-24	3.046450E-24
	m	EKF	3.80244608	9.70e-07	6.40E-02	2.001257E-04	2.062310E-04
10000 avalas	111	UKF	3.80244977		6.50E-02	1.961237E-04	2.021250E-04
10000 Cycles	C	EKF	1.50024650e-10	1.042.06	1.60E-02	2.693654E-25	2.699850E-25
	C	UKF	1.50024806e-10	1.046-00	1.70E-02	2.639787E-25	2.645940E-25
	m	EKF	3.80020006	2 022 06	5.30E-03	8.986668E-05	8.990750E-05
20000 avalas	111	UKF	3.80020774	2.028-00	5.50E-03	8.806974E-05	8.811290E-05
50000 Cycles	C	EKF	1.50014279e-10	2 540 06	9.50E-03	1.175416E-25	1.177500E-25
	C	UKF	1.50014660e-10	2.546-00	9.80E-03	1.151919E-25	1.154070E-25
	m	EKF	3.80012911	1 140 06	3.40E-03	3.232599E-05	3.234300E-05
60000 avalas	111	UKF	3.80013344	1.14e-00	3.50E-03	3.167990E-05	3.169780E-05
oooo cycles	C	EKF	1.50011431e-10	2 160 06	7.60E-03	6.206314E-26	6.219650E-26
	C	UKF	1.50011950e-10	3.408-00	8.00E-03	6.082333E-26	6.096610E-26

The comparisons between $\{m, \hat{m}_k\}$ and $\{C, \hat{C}_k\}$ given by EKF and UKF as well as 90% C.I. for one run of these recursive algorithms at every cycle k are respectively illustrated in Fig.7 and Fig.8. The curves of EKF and UKF almost overlap, which indicates the state-parameter estimates of these two algorithms are in close vicinity of one another. This agrees with the 5th column in Table 6 that the difference between EKF and UKF is minor. The C.I. curves of EKF and UKF can also hardly be distinguished by naked eye.

Crack length evolution based on one simulation is illustrated in Fig.9 (a) and the first 10000 cycles is presented in Fig.9 (b) in order to observe more clearly the difference between the true crack size and the estimation. UKF once again perform comparably to EKF. Good agreement can be seen as well between the true values and the output of these recursive methods (EKF/UKF). However, note that UKF is much more time-consuming than that of EKF (12.38s vs 3.48s, on an Intel(R) i3-4130, 3.40GHz CPU) since a transformation of 7 sigma points is required at each time step compared to only one point performed in EKF.

Some researchers argued that UKF outperform EKF on problems which have a strong nonlinearity [19,20,41]. The reason for the very similar performance of the two estimators may be due to the minor nonlinearity of the system: under the considered initial condition, the damage propagation rate does not exhibit a strong nonlinearity. In order to test these two methods in a more nonlinear scenario, we increased the initial crack length value $a_0=1mm$ and kept other condition unchanged, in which case the damage increase rate clearly presents an exponential growth trend. The comparison of $\hat{\Theta}_k$, Θ_{error} , MSE and MSE are shown in Table 7. The same conclusion that no great difference exists between EKF and UKF can be drawn compared with the 'minor nonlinear' case. The parameter and crack length comparisons are illustrated in Figs. 10-12. Once again the curve of UKF and EKF are almost overlapping and difficult to distinguish, which indicate that UKF results are comparable in accuracy to EKF and do not depend here on the nonlinearity of the problem.



Fig.9 Comparison of crack length evolution in minor nonlinear case based on one simulation, measurement points are plotted every 100 cycles

	^	^						
Table 7	Θ / Θ	IMCE	MCE and	Course	50 aimulat	ana in	strong nonlin	
rable /	$\Theta_{1} / \Theta_{mmm}$	INISE	INSE M and	i C Over	50 siniulau	ons m	strong nomin	lear case
	K error						0	

No. of cycles	Para.	Filter	$\hat{\mathbf{\Theta}}_k$	Difference	Θ_{error} (%)	MŜE	MSE
	m	EKF	3.79411085	3 220 07	1.6E-01	1.603800E-02	1.607300E-02
100 avalas	111	UKF	3.79410963	3.23e-07	1.6E-01	1.571700E-02	1.575200E-02
100 cycles	C	EKF	1.50422797e-10	6 80 0 08	2.8E-01	2.845485E-23	2.863725E-23
	C	UKF	1.50422786e-10	0.898-08	2.8E-01	2.788576E-23	2.806451E-23
		EKF	3.80425106	3.24e-06	1.1E-01	1.567000E-03	1.585000E-03
1000 avalas	III	UKF	3.80423872		1.1E-01	1.536000E-03	1.554000E-03
1000 cycles	C	EKF	1.49580266e-10	(22- 07	2.8E-01	1.567000E-03	4.126856E-24
	C	UKF	1.49580172e-10	0.32e-07	2.8E-01	3.868153E-24	4.044409E-24
		EKF	3.80188861	$2.91_{2.05}$	5.0E-02	1.356330E-04	1.392726E-04
10000 cycles	III	UKF	3.80178170	2.816-03	4.7E-02	1.329027E-04	1.360771E-04
	С	EKF	1.49861313e-10	5.17e-06	9.3E-02	2.343272E-25	2.539537E-25



Fig.12 Comparison of crack length evolution in strong nonlinear case based on one simulation, measurement points are plotted every 100 cycles

4.4 Effect of measurement noise distribution

In order to assess whether the proposed Kalman filtering framework will maintain its advantages when using the measurements with other type of noise, we conducted an additional study on the strong nonlinearity case (i.e., the true initial crack length $a_0=1mm$) to test the robustness of our approaches considering a different noise distribution. We simulated the measurements data according to the true crack size to which a uniformly distributed noise was added instead of a normally

one. The half range of the added uniformly distributed noise corresponds to 3 sigma of the normal measurement noise distribution given in Table 4. Note that except the use the uniformly distributed measurement noise, the rest experiment conditions are the same with what we have done in the strong nonlinear case in section 4.3. The results are given in Table 8 and Figs.13-15. Comparing these results with Table 7 and Figs.10-12, we found that when measurements with uniform noise are used, the crack length as well as {m, C} converge to their true values between 47% and 37% slower than when normal measurement noise was considered. This convergence is still considered acceptable in the present context since as show Figures 13 and 14 the convergence is rather quick with respect to the total lifecycle of the aircraft. Accordingly we conclude that for this application EKF and UKF are both relatively robust.

No. of cycles	Para.	Filter	$\hat{\Theta}_k$	Difference	Θ_{error} (%)	MŜE	MSE
		EKF	3.801701752		4.48E-02	4.173752E-02	4.174047E-02
100 1	m	UKF	3.801699993	1,76E-06	4.47E-02	4.090246E-02	4.090535E-02
100 cycles	C	EKF	1.49673143E-10		2.18E-01	6.756016E-23	6.766918E-23
	C	UKF	1.49673128E-10	1,49E-17	2.18E-01	6.620895E-23	6.631579E-23
		EKF	3.784815397		4.00E-01	5.299447E-03	5.534725E-03
1000 1	m	UKF	3.784802902	1,25E-05	4.00E-01	5.193028E-03	5.423980E-03
1000 cycles	C	EKF	1.50284818E-10		1.90E-01	8.000934E-24	8.083710E-24
	C	UKF	1.50284723E-10	9,52E-17	1.90E-01	7.840865E-24	7.921932E-24
		EKF	3.797685854		6.09E-02	3.750411E-04	3.805057E-04
10000 cycles C	m	UKF	3.801699993	-4,01E-03	6.37E-02	3.674089E-04	3.732651E-04
	C	EKF	1.50177782E-10		1.19E-01	9.673154E-25	9.995668E-25
	С	UKF	1.50177018E-10	7,64E-16	1.18E-01	9.479697E-25	9.793052E-25
		EKF	3.799745124		6.71E-03	1.644527E-05	1.651156E-05
20000 1	m	UKF	3.799694034	5,11E-05	8.05E-03	1.611633E-05	1.620995E-05
30000 cycles	C	EKF	1.50069748E-10	,	4.65E-02	2.447189E-25	2.496831E-25
	C	UKF	1.50069692E-10	5,68E-17	4.65E-02	2.398290E-25	2.446860E-25
		EKF	3.799861762	,	3.64E-03	2.334348E-06	2.353848E-06
60000 cycles	m	UKF	3.799865116	-3,35E-06	3.55E-03	2.287669E-06	2.305863E-06
	a	EKF	1.50060122E-10	,	4.01E-02	1.324114E-25	1.360999E-25
	С	UKF	1.50060853E-10	-7,31E-16	4.06E-02	1.297684E-25	1.334715E-25
:	5 ₁		T	3.9		True	
				3.88-		-Mean of EKF estin	nate
			Mean of UKF estimate			Mean of UKF estin	nate
4.5	5		90% C.I. of UKF	3.86-		90% C.I. of UKF	
				3.84	been and the second		
4	4-			3.82	ore colocome		-
E			000000000000000000000000000000000000000	E 3.8		Conceptore and a conceptor and	2000
3.5	5-			3.78		00000	-
				3 76-			
				2.74			
	2			3.74			1
				3.12			1
2.5	0 1	2	3 4 5	3.7 <u> </u>	1 2	3 4 5	6
		Flig	gnt Cycles x 10	ŧ	Flig	nt Cycles	x 10 ^⁴

Table 8 $\hat{\Theta}_{k} / \Theta_{error} / MSE / MSE m$ and C over 50 simulations in strong nonlinear case, using uniform measurement noise





Fig.14 Comparison of convergence process of C in strong nonlinear case, using uniform measurement noise



Figure 15 Comparison of crack length evolution in strong nonlinear case based on one simulation, using uniform measurement noise, measurement points are plotted every 100 cycles

5. Conclusions

In this paper, estimation for Paris' law constants and crack length evolution has been formalized as a nonlinear filtering problem. Two filtering approaches, Extended Kalman filter and Unscented Kalman filter, have been applied to determine the Paris' law constants and the fatigue crack length behavior. Our results indicate that both these two methods identify Paris' law constants and estimate the crack length fairly well. For the purpose of state-parameter identification with Paris' law, EKF has a comparable accuracy to UKF while it is less expensive in terms of computational demand, in both minor nonlinear and strong nonlinear situations. The UKF algorithm is theoretically superior to EKF but in practical, it does not show obvious improvement on the problem considered here. In addition, UKF is sensitive to the scaling parameter κ and has a strict constraint for the choice of the initial state error covariance matrix P_0 . Improper selection of κ will cause estimator divergence while an inappropriate value of P_0 may lead to a non-positive semi definite state error covariance matrix after several times of iteration, which makes Cholesky factorization impossible. Accordingly, we conclude that EKF is an accurate and efficient recursive state/parameter estimation approach for fatigue crack propagation problem. Future work involves the use of EKF in condition-based maintenance through the computation remaining useful life.

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