7th World Congresses of Structural and Multidisciplinary Optimization
COEX Seoul, 21 May – 25 May 2007, Korea

Applying dimensional analysis to response surface methodology to reduce the number of design variables

Christian Gogu[1,2], Raphael Haftka[1], Satish Bapanapalli[1], Bhavani Sankar[1]

[1] Department of Mechanical and Aerospace Engineering, University of Florida
PO Box 116250, Gainesville, FL 32611-6250, USA
[2] Centre Sciences des Matériaux et des Structures (SMS-MEM), Ecole Nationale Supérieure des Mines de St. Etienne, 42023 St. Etienne cedex 2, France
Corresponding author's e-mail: cgogu@ufl.edu

Abstract
The design of an integrated thermal protection system (ITPS) for spacecraft reentry involves thermal as well as structural constraints. In the present paper we focus on the thermal constraint represented by the maximum temperature of the bottom face of the ITPS panel, which is not allowed to exceed a critical value. A response surface approximation (RSA) of this temperature was needed in order to reduce optimization computational time. The finite element model used to evaluate the maximum temperature at the design of experiment points involved a total of 15 parameters of interest for the design: 9 thermal material properties and 6 geometric parameters of the ITPS model. In order to reduce the dimensionality of the response surface approximation, dimensional analysis was utilized. A small number of assumptions simplified the equations of the transient thermal problem allowing easy nondimensionalization using classical techniques. The nondimensional equations together with a global sensitivity analysis showed that the maximum temperature mainly depends on only two nondimensional parameters which were selected to be the design variables for the RSA. It is important to note that the RSA was still constructed using the accurate finite element model which does not employ any of the simplifying assumptions used to determine the nondimensional parameters. The two variable RSA was checked for its accuracy in terms of geometric parameters and material properties variables at 855 additional test points using the finite element model. The error in the RSA was not due to the quality of the fit but mainly due to the reduction from 15 to only two variables. This error had an average value of 2.31 K, the standard deviation of the error was 2.01 K and the maximum error among the 855 points was found to be 14.7 K (for a range of the temperature RSA of about 250 K). The two dimensional nature of the RSA allowed its graphical representation, which was used for material comparison and selection for the ITPS among hundreds of possible materials.

Keywords: response surface approximation, surrogate, dimensionality reduction, dimensional analysis, thermal design

1. Introduction

Throughout the past century dimensional analysis has been an extremely successful tool for solving scientific and engineering problems and for presenting results in a compact form. The first theoretical foundations of dimensional analysis were set by Vashy [1] and Buckingham [2] at the end of the 19th century. Since then and up to the 1960s nondimensional solutions, often in the form of graphs in nondimensionalized variables, have been among the major forms of transmitting knowledge among scientists and engineers. Indeed before the advent of the computer era, developing solutions to differential equations, modeling physical processes, was a long, complex task. When a solution to a physical problem was found it was published in its nondimensional form in order to make it applicable to the largest range of problems.

This approach changed with the advent of widely available numerical simulation software and hardware starting in the 1960s. Under these new circumstances it becomes indeed much easier to obtain a solution directly to the problem one is interested in, without having to go through dimensional analysis and seek published solutions of the nondimensional equations. Numerical computer simulation gained huge popularity and dimensional analysis was slowly left behind except in areas where nondimensional parameters have a strong physical interpretation and allow us to differentiate between regimes of different numerical solution techniques (Mach number, Reynolds number etc.).

With the increase in computational power, numerical simulation became not only feasible for single engineering analyses but it became increasingly possible to use these numerical techniques such as finite element (FE) in design optimization, often in conjunction with the use of surrogate models. One issue that appeared with the use of surrogate models is the "curse of dimensionality" which increases the number of experiments needed for a surrogate exponentially with the number of dimensions for a same accuracy.

One way of reducing the dimensionality of the surrogate is by applying dimensional analysis to the physical problem the FE model and thus the surrogate is describing. Several studies looked at the advantages of using dimensional analysis in conjunction with surrogates or response surface approximations. References [3]-[5], all show that by using intrinsic or nondimensional variables which are characteristic of the problem, better accuracy of the RSA can be obtained. For the same number of numerical simulations the reduction in the number of variables associated with dimensional analysis allows a fit with a higher order polynomial. Among the references cited none used FE to construct the RSA. Reference [4] used statistical data from a survey while reference [5], while evoking the possibility of using FE, used simple analytical objective functions to illustrate the advantages of the approach in terms of improving the accuracy of the RSA and reducing the number of experiments.

In [6] Venter and Haftka gave an illustration of how dimensional analysis was directly used to reduce the number of variables of a RSA constructed from a FE buckling model of a plate with an abrupt change in thickness. The dimensional analysis was done directly on the governing equations and the boundary conditions solved by the FE. The approach reduced the number of variables of the RSA from nine to seven. However nondimensionalzing directly the governing equations and boundary conditions of a problem is not always easy and can introduce a significant number of nondimensional parameters, of which some might only have marginal influence on the design problem considered. Instead we may be able to discover the most important parameters from a simplified version of the equations. Applying dimensional analysis to this simplified problem allows to further reduce the number of parameters while only keeping the ones relevant to the design task considered.

In the present paper we illustrate this approach on the example of a spacecraft atmospheric reentry transient thermal problem involving the maximum temperature during reentry. Dimensional analysis on a simplified problem in conjunction with a global sensitivity analysis allowed to reduce the number of variables that determined the maximum temperature from 15 parameters to 2 nondimensional parameters. A response surface approximation (RSA) of this temperature was constructed using a FE model, that doesn't use any of the previous simplifications, and the ability of the 2 dimensional RSA to represent all of the 15 initial variables was tested.

In Section 2 we describe the thermal problem of atmospheric reentry and in Section 3 the FE model used. Then in Section 4 we use dimensional analysis on a simplified version of the thermal problem together with a global sensitivity analysis to determine the minimum number of nondimensional variables for the maximum temperature RSA. In Section 5 we fit the data obtained from the accurate, non-simplified FE model with a response surface approximation function of the previously determined nondimensional parameters. We then compare in Section 6 the accuracy of the RSA with additional test points using the FE model. Finally we give in Section 7 a brief overview of how the RSA was used to carry out a materials selection for the design and optimization of an integrated thermal protection system (ITPS). Section 8 presents concluding remarks.

2. Thermal problem of atmospheric reentry

An integrated thermal protection system (ITPS) is a proposed spacecraft system fulfilling both thermal protections requirements during reentry and structural requirements during all the phases of the mission. Thus in such a system the thermal protection function would be integrated with the structural function of the spacecraft. Our study involves an ITPS based on a corrugated core sandwich panel construction. The design of such an ITPS involves both thermal and structural constraints. In the present paper we focus on the thermal constraint represented by the maximum temperature of the bottom face sheet (BFS) of the ITPS panel. The combined thermo-structural approach is presented in a separate article [7]. A response surface approximation (RSA) of this maximum BFS temperature was needed in order to reduce optimization computational time.

In order to construct the maximum BFS temperature RSA we developed a finite element model using the commercial FE software Abaqus. The corrugated core sandwich panel design as well, as the thermal problem of atmospheric reentry is shown in Figure 1. The ITPS panel is subject to an incident heat flux assumed to vary as shown in Figure 2. This heat flux is typical of a reusable launch vehicle (RLV). The general approach presented in the present paper would apply for different heat flux profiles.

Radiation is also modeled on the top face sheet (TFS) with a relative emissivity of 0.8, which is typical for TPS exterior surfaces [8]. The BFS is assumed perfectly insulated, which is a worst case assumption, since if heat could leak through the BFS the maximum temperature would decrease, becoming less critical. The core of the sandwich panel is assumed to be filled with Saffil foam insulation, while we will explore different materials for the three main sections (TFS, BFS and Web), materials of which we want to determine the combination leading to an ITPS with the lowest maximum BFS temperature.

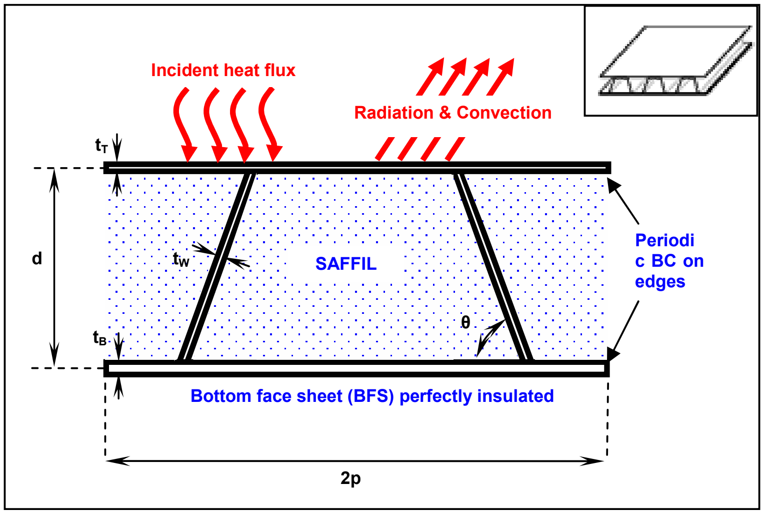

Figure 1: Corrugated core sandwich panel design with representation of the thermal problem and geometric parameterization

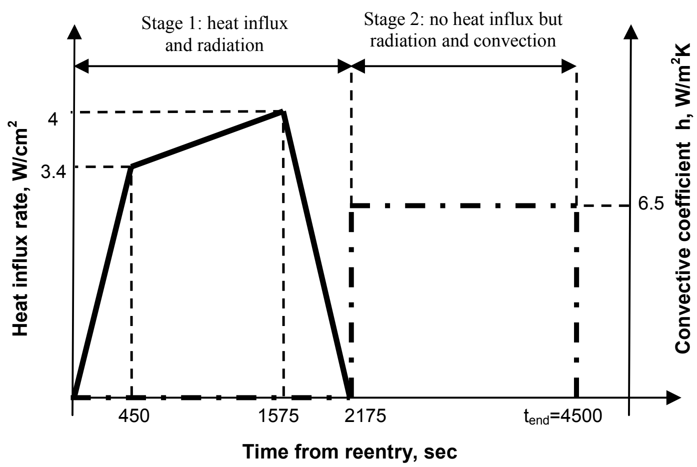

Figure 2: Incident heat flux (solid line) and convection (dash dot line) profile with reentry time on the TFS surface

3. Finite element model of the thermal problem

The thermal problem just described is modeled using the Abaqus FE software. The problem is modeled as a one dimensional heat transfer analysis as represented in Figure 3. The core of the sandwich panel has been homogenized using the rule of mixtures formulae given below:

$$\rho = \frac{\rho_W V_W + \rho_S V_S}{V} = \frac{\rho_W t_W + \rho_S (p\sin\theta - t_W)}{p\sin\theta} \tag{1}$$

$$C = \frac{C_W \rho_W V_W + C_S \rho_S V_S}{\rho V} = \frac{\rho_W C_W t_W + \rho_S C_S (p\sin\theta - t_W)}{\rho_W t_W + \rho_S (p\sin\theta - t_W)} \tag{2}$$

$$k = \frac{k_W A_W + k_S A_S}{A} = \frac{k_W t_W + k_S (p\sin\theta - t_W)}{p\sin\theta} \tag{3}$$

where ρ stands for density, C for specific heat, k for conductivity, t for thickness, θ for the corrugation angle and p for the length of a unit cell (cf. Figure 1). The subscripts *W* and *S* represent structural Web material and Saffil foam, respectively, while no subscript represents the homogenized core. *A* stands for area of cross-section through which the heat flows and *V* for the volume of each section.

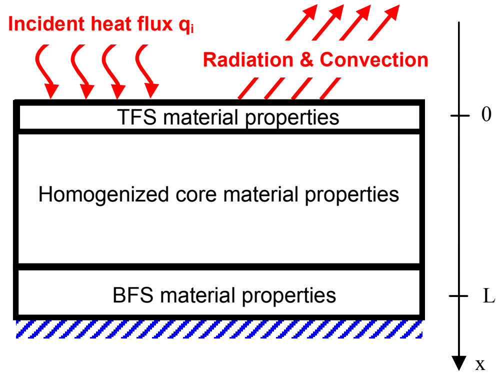

Figure 3: 1D heat transfer model representation using homogenization (representation not to scale)

It has been shown in reference [7] that such a one dimensional FE model can accurately predict the temperature distribution through the thickness of the sandwich panel, the maximum difference in temperature prediction between the 1D model and a 2D model being less than 20K. For this preliminary design phase of the ITPS this difference is acceptable and a 1D model is used in the next parts for the thermal analyses. Radiation, convection and the incident heat flux (as shown in Figure 1 and 2) were modeled in the Abaqus 1D model using four steps (three for stage one of Figure 2 and one for stage two). 54 three node heat transfer link elements were used in the transient analyses.
For this one-dimensional thermal model the governing equations and boundary conditions can be written as following:

Heat conduction equation: $$\frac{\partial}{\partial x}\left(k(x,T)\frac{\partial T(x,t)}{\partial x}\right) = \rho(x,T)C(x,T)\frac{\partial T(x,t)}{\partial t} \quad (4)$$

Initial condition: $$T(x,t=0) = T_i \quad (5)$$

Boundary conditions: $$q(x=0,t) = -k_{TFS}\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0} = q_i(t) - \varepsilon T(0,t)^4 - h(t)T(0,t) \quad (6)$$

$$q(x=L,t) = 0 \quad (7)$$

where ρ is the density, k the thermal conductivity and C the specific heat of the ITPS panel. T_i is the initial temperature of the panel before atmospheric reentry, ε the emissivity of the TFS while $q_i(t)$ is the heat influx and $h(t)$ the convection coefficient at the TFS, which are time dependent, varying as shown in Figure 2. We note that most of the material properties are temperature dependent and due to the different materials in the different ITPS sections most material properties also depend on the position x. If we would have nondimensionalized directly the equations, due to these dependencies we would have obtained a high number of nondimensional parameters. To reduce to a minimum the number of parameters of the response surface the thermal problem will be studied in the next section under several assumptions which allow easier nondimensionalization of the equations as well as leaving out variables which have only little effect.

4. Determining the minimum number of parameters for the temperature response surface

4.1 Simplifying the problem

One of our aims is to determine which materials are the best for use in the ITPS panel. The thermal model presented in the previous section involves 13 material parameters (specific heat C_i, conductivities k_i and densities ρ_i of the TFS, BFS, Web and Saffil as well as the emissivity ε of the TFS) of which most are temperature dependent. Some of these parameters were fixed, including ε as well as all the foam parameters (the foam material has been fixed to Saffil, which has been determined in previous studies [9] and [10] as being the best suited foam for use in similar metallic thermal protection systems). Concerning the emissivity of the TFS, it depends more on surface treatments applied than on the nature of the TFS material (thus a typical value for this kind of application of 0.8 was used [8],[10]). Fixing these parameters leaves 9 variables to come from material selection. Describing temperature dependency of the parameters would increase this number further.

In order to avoid an unnecessarily high dimensional space we wanted to reduce to a minimum the number of material parameter design variables. For this purpose, the heat transfer equations were studied under several simplifying assumptions that allowed not having to consider for the material selection process, material parameters that have a negligible effect on the maximum BFS temperature. The following assumptions have been established and checked on a Nextel(TFS)-Zirconia(Web)-Aluminum(BFS) ITPS having the dimensions shown in Table 1. These dimensions were found in reference [7] to be the optimal geometry for an Inconel(TFS)-Ti6Al4V(Web)-Al(BFS) material combination (for details cf. reference [7]).

Parameter	t_T (mm)	t_B (mm)	t_W (mm)	θ (deg)	d (mm)	p (mm)
Value	2.1	5.3	3.1	87	120	117

Table 1: Dimensions of the ITPS used among other to establish the simplifying assumptions. These dimensions were optimal for an Inconel(TFS)-Ti6Al4V(Web)-Al(BFS) ITPS (cf. [7])

Considering the nature of the problem, following assumptions could be made:
1.) The three thermal properties of the TFS (C_{TFS}, k_{TFS} and ρ_{TFS}) have negligible impact on the maximum BFS temperature, mainly due to the small thickness of the TFS (about 2.2mm compared to a total ITPS thickness of about 120mm). This allowed removing C_{TFS}, k_{TFS} and ρ_{TFS} from the relevant parameters influencing the BFS temperature.
2.) The temperature is approximately constant through the BFS, because the BFS thickness is small (typically 5mm thick compared to a total ITPS thickness of 120mm) and its conductivity is about one order of magnitude higher than that of the homogenized core. This allowed removing k_{BFS} from the relevant parameters and simplifying the boundary condition at the BFS.
3.) In order to avoid having a large number of material properties variables several assumptions were made concerning the temperature dependency of these material properties. In the FE model temperature dependency has been included for all materials but the largest temperature dependency was that of the Saffil foam. So for the TFS, Web and BFS materials constant properties were chosen as for Saffil the material properties were assigned the values at a representative temperature chosen such as to minimize the difference between the maximum BFS temperature when using the constant values and the one when using temperature dependent values. The representative temperature was first established on the ITPS design in Table 1 with a Nextel(TFS)-Zirconia(Web)-Aluminum(BFS) material combination and the effects of varying the materials were then found to be small enough to use this constant value for the range of materials we consider.

It is important to note that these simplifying assumptions were used only in order to determine the minimum number of variables for the RSA, but the finite element analysis used to construct the experiments and validate the RSA did not incorporate any of these simplifying assumptions but used the accurate FE model presented in section 3. Furthermore, the success of the dimensional analysis does not depend on the assumptions leading to small errors. As long as the RSA that will be constructed changes approximately the same way as the exact temperature, a close to constant distance between them will not lead to significant errors since the RSA is fitted to the accurate FE experiments.

These assumptions allowed to reduce the number of relevant material parameters influencing the maximum BFS temperature from 9 to 5: we now only have the density and specific heat of the BFS and Web as well as the conductivity of the Web. These assumptions also allowed to simplify the problem so that it can be easily nondimensionalized as will be shown next.

4.2 Nondimensionalizing the thermal problem

Under the previous simplifying assumptions the thermal problem is equivalent to the one shown in Figure 4 and its equations can be rewritten as follows.

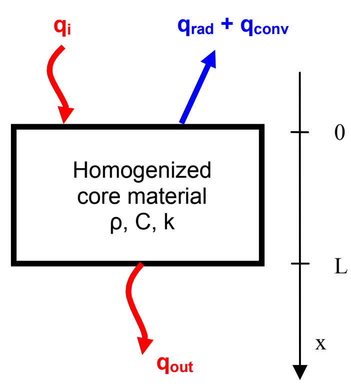

Figure 4: Simplified thermal problem for dimensional analysis

Heat conduction equation: $k\dfrac{\partial^2 T(x,t)}{\partial x^2} = \rho C \dfrac{\partial T(x,t)}{\partial t}$ for $0<t<t_{end}$ (9)

Initial condition: $T(x,t=0) = T_i$ (10)

Boundary conditions: $q_{out} = -k\left.\dfrac{\partial T(x,t)}{\partial x}\right|_{x=L} = \rho_B C_B L_B \left.\dfrac{\partial T(x,t)}{\partial t}\right|_{x=L}$ (11)

$$q_{in} = -k\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0} = q_i(t) - \varepsilon T(0,t)^4 - h(t)T(0,t) \qquad (12)$$

where L and L_B are the thicknesses of the homogenized core and the BFS respectively; t_{end} is the duration of the heat influx; ρ, C and k are the density, specific heat and conductivity of the homogenized core; ρ_B and C_B those of the BFS.

In order to nondimensionalize the equations of this problem we use the Vashy-Buckingham theorem (or Pi theorem) [1,2] to know the minimum number of nondimensional variables. The Vashy-Buckingham theorem states that we have to count the total number of variables and the corresponding number of dimensional groups. In our case we have the following:

Variable	T	T_i	x	L	t	t_{end}	k	ρC	$\rho_B C_B L_B$	q_i	ε	h
Unit	K	K	m	m	s	s	$\frac{W}{m \cdot K}$	$\frac{W \cdot s}{m^3 \cdot K}$	$\frac{W \cdot s}{m^2 \cdot K}$	$\frac{W}{m^2}$	$\frac{W}{m^2 \cdot K^4}$	$\frac{W}{m^2 \cdot K}$

We have a total of 12 variables in 4 dimensional groups (m, s, K, W). From the Vashy-Buckingham theorem we know that we can have a minimum of 12 – 4 = 8 nondimensional groups. We now have to determine the precise expressions of these parameters. By manually transforming the equations (9)-(12) of the thermal problem, the following non dimensional parameters have been constructed:

$\dfrac{T}{T_i} = \Gamma$ (13); $\dfrac{x}{L} = \xi$ (14); $\dfrac{t}{t_{end}} = \tau$ (15); $\dfrac{k t_{end}}{L^2 \rho C} = \beta$ (16);

$\dfrac{L_B \rho_B C_B}{L \rho C} = \gamma$ (17); $\dfrac{L \varepsilon T_i^3}{k} = \kappa$ (18); $\dfrac{L q_i(t)}{k T_i} = \varphi(\tau)$ (19); $\dfrac{h(t) L}{k} = Bi(\tau)$ (20)

In terms of these nondimensional variables the thermal problem can be written in the following nondimensional form:

Heat conduction equation: $\beta \dfrac{\partial^2 \Gamma}{\partial \xi^2} = \dfrac{\partial \Gamma}{\partial \tau}$ for $0<\tau<1$ (21)

Initial condition: $\Gamma(\xi, \tau = 0) = 1$ (22)

Boundary conditions: $-\beta \left.\dfrac{\partial \Gamma}{\partial \xi}\right|_{\xi=1} = \gamma \left.\dfrac{\partial \Gamma}{\partial \tau}\right|_{\xi=1}$ (23)

$$-\left.\frac{\partial \Gamma}{\partial \xi}\right|_{\xi=0} = \varphi(\tau) - \kappa \cdot \Gamma(0,\tau)^4 - Bi(\tau)\cdot\Gamma(0,\tau) \qquad (24)$$

The complete set of nondimensional variables needed for the considered problem is that given in equations (13)-(20). The non dimensional temperature Γ can be expressed function of the nondimensional distance ξ and the nondimensional time τ as well as function of five other non dimensional parameters. Since at the maximum BFS temperature we are at a fixed location and we are not interested in the time at which this maximum occurs, the non dimensional distance ξ and the nondimensional time τ ill not have to intervene in the maximum BFS temperature RSA.

The physical interpretation of the other five nondimensional parameters given in equations (16)-(20) is the following. β, the Fourier number or a nondimensional thermal diffusivity, is the ratio between the rate of heat conduction and the rate of heat storage (thermal energy storage) of the homogenized core. γ is the ratio between the heat capacity of the BFS and heat capacity of the homogenized core, κ the ratio between the rate of radiation and the rate of heat conduction. φ is the ratio between the incident heat flux and the rate of heat conduction, or can be seen as a nondimensional heat flux. Finally Bi, the Biot number, is the ratio between the rate of convection and the rate of heat conduction.

We can notice that the three nondimensional parameters κ, φ and Bi are all proportional to L/k while all the other parameters in κ, φ and Bi are fixed in our study. Indeed we are only interested in varying the materials and the geometry but the initial temperature T_i, the emissivity ε, the incident heat flux profile $q_i(t)$ and the convection film coefficient profile $h(t)$ are all fixed in the present study (cf. Figure 2 for the profiles of $q_i(t)$ and $h(t)$ used). This means that for our purpose we can consider only one of these three nondimensional parameters: κ for example.

Summing up, the dimensional analysis allowed us to determine that we can construct a response surface approximation of the maximum BFS temperature function of the three parameters β, γ and κ. However after we constructed a first RSA in these three parameters it seemed that the maximum BFS temperature is relatively insensitive to κ. To check this we conducted a global sensitivity analysis, using Sobol's approach [11]. We found out that variable β accounts for 35.7% of the model variance, variable γ accounts for 64.1% of the model variance while variable κ accounts for only 0.06% of the model variance. This means that we can explain almost the entire behavior of the model with only the two variables β and γ.

From a physical point of view the fact that κ has a negligible role can be explained as follows: κ is proportional to L/k which is also present in β. That means that if we want to change κ while keeping β constant we need to also modify t_{end} (which is the only other variable in β that does not intervene neither in γ nor in κ). If we increase κ by decreasing k we need to also increase t_{end} by a certain amount to keep β constant. Decreasing k has the effect of lowering the BFS temperature while increasing t_{end} has the effect of making it higher. From the global sensitivity analysis it turns out that these two effects cancel each other out which explains why κ has very little impact.

5. Maximum BFS temperature RSA

A response surface approximation (RSA) in the two nondimensional parameters β and γ was now constructed. The RSA was needed in order to reduce the computational cost for the comparison of different material combinations. Indeed there are about 30,000 possible material combinations, which would have taken about 2500 hours if each would have had to be evaluated using the Abaqus FE model. Instead using the response surface approximation allows to greatly reduce the computational time needed (to about 6 hours).

We chose as design of experiments (DoE) a simple 11x11 grid. The boundaries for each variable are given in Table 2 (the bounds were chosen so that all material combinations under consideration fell inside these values for the geometry values fixed in Table 1). The choice of a grid was done based on the relatively small computational time needed for each simulation (about 3min). Furthermore an 11x11 grid, which is a relatively dense DoE, allows a good quality of a fit thus leaving as the major remaining error in the RSA only that coming from the reduction from 15 to 2 variables through the nondimensionalization on the simplified thermal problem. We settled on an 11x11 grid after comparing the RSA obtained with this grid with an RSA obtained with a 6x6 grid. Comparison at 200 latin hypersquare points of these two RSAs yielded a mean value of the difference of only 0.1%.

Variable	β	γ
Range	0.1-0.5	0.6-2.4

Table 2: Ranges of the non dimensional design variables

The model used for the computation of the maximum BFS temperature for each experiment is the Abaqus 1D model described in section 3, which does not use any of the simpliflying assumptions of section 4. Abaqus was coupled with Matlab for an automated run through all the experiments. The maximum BFS temperatures resulting from the DoE was fitted with a cubic spline approximation. The response surface of the maximum BFS temperature function of β and γ is represented in Figure 5. One of the advantages of having only two variables in the RSA is an easy graphical representation of the results. This graphical representation possibility will be later used for the material selection as well.

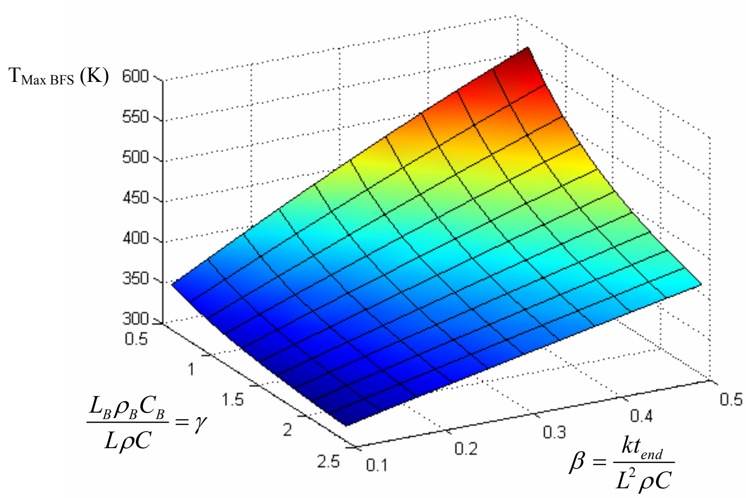

Figure 5: Maximum BFS temperature two variables response surface

6. Accuracy of the RSA

The two variable RSA accounts for a total of 15 parameters: the nine thermal material properties (C_i, k_i, ρ_i) as well as for the six geometric parameters of the ITPS panel (t_i, p, d, θ) as is shown in equations (25) and (26). These equations were obtained by substituting back the expressions of ρ, C and k from equations (1)-(3) into the equations (16) and (17) of the nondimensional parameters β and γ.

$$\beta = \frac{[k_W t_W + k_S (p\sin\theta - t_W)]\cdot t_{end}}{(d - 0.5t_T - 0.5t_B)^2 \cdot [\rho_W C_W t_W + \rho_S C_S (p\sin\theta - t_W)]} \tag{25}$$

$$\gamma = \frac{t_B \rho_B C_B p\sin\theta}{(d - 0.5t_T - 0.5t_B)\cdot [\rho_W C_W t_W + \rho_S C_S (p\sin\theta - t_W)]} \tag{26}$$

To test the accuracy of the RSA we compared it with the accurate FE analyses that do not involve any of the simplifying assumptions made for nondimensionalization at 855 latin hypersquare points spread in the 15-dimensional variables space (9 materials properties and 6 geometric parameters) with the bounds given in Table 3. The mean of the absolute difference was 2.31K with a standard deviation of 2.01K, while the maximum difference observed among the 855 points was 14.7K (the range of the RSA is about 250K).

	k_{Web}	ρ_{Web}	C_{Web}	k_{BFS}	ρ_{BFS}	C_{BFS}	k_{TFS}	ρ_{TFS}	C_{TFS}	t_T	t_B	t_W	θ	d	p
LB	2	2500	500	2	1550	900	3	2000	500	1E-3	4.6E-3	2.24E-3	80	0.102	0.099
UB	7	6000	950	50	3000	1820	50	5000	1700	3.7E-3	7E-3	4.16E-3	90	0.150	0.150

Table 3: Lower bounds (LB) and upper bounds (UB) used for sampling in the 15 variables space. All units are SI.

It can be noticed that the mean of the absolute error is low, however in some cases the error can be much higher, the maximum error being about 6 times higher than the mean. To gain more insight of where the maximum errors occur anti-optimization [12,13] of the error in the RSA was carried out. The anti-optimization process looks to find the places with the highest error in the RSA and by looking at the designs corresponding to the antioptimum we can understand what causes these errors. Antioptimizations with a fixed material combinations and the geometry allowed to vary within the bounds given in Table 3 were carried out and showed that the RSA has poor accuracy when the geometry is far away from the one for which the representative temperature of assumption 3.) (cf. section 4.1) was established. For these unusual geometries the representative temperature shifts due to temperature dependence of the core; this shift is not accounted for by the RSA which explains the poor accuracy for these geometries. If we wanted to improve the accuracy of the RSA for a large range of geometries we would have to better account for the temperature dependence of the core by introducing additional nondimensional parameters. For the geometry for which we will use the RSA in the next section however the maximum error was 7.6 K, which was considered acceptable for the purpose of material selection.

7. Applying the RSA for comparison of materials for the ITPS

The easy graphical representation of the two dimensional RSA was used next for comparison of alternate materials for the ITPS sections. We seek materials providing a low maximum BFS temperature. The dimensions of the ITPS are once again fixed to the values in Table 1.

In order to have an exhaustive search, the material database software CES Selector 2005 EduPack by Granta Design was used. Several constraints on materials properties were imposed during the search in the database, constraints on properties such as maximum service temperature, fracture toughness and Young's modulus, in order to avoid unreasonable materials. These constraints reduced the number of possible materials to 235 for the BFS and 127 for the Web.

To compare the Web materials, the BFS material was fixed to Aluminum alloy 2024 and the potential Web materials were plotted in the (β, γ) plane with the contours of the maximum temperature superposed as shown in Figure 7, allowing comparison of the different mateials with respect to the corresponding maximum BFS temperature. This figure shows that materials such as alumino-silicate/Nextel 720 composites or Zirconia ceramics provide a significant reduction in the maximum BFS temperature compared to metals such as titanium alloys, which were considered in previous designs (cf. [7]). Accordingly alumino-silicate/Nextel 720 composites and Zirconia were selected for further study as good potential candidates for the Web section of the ITPS panel.

A complete materials comparison and selection study using this approach is presented in reference [14]. This reference presents selection of potential materials for all of the three sections (TFS, Web and BFS) and also includes an optimization of the geometry of the ITPS panel with respect to mass per unit area, leading to a ranking of the different material combinations for the ITPS.

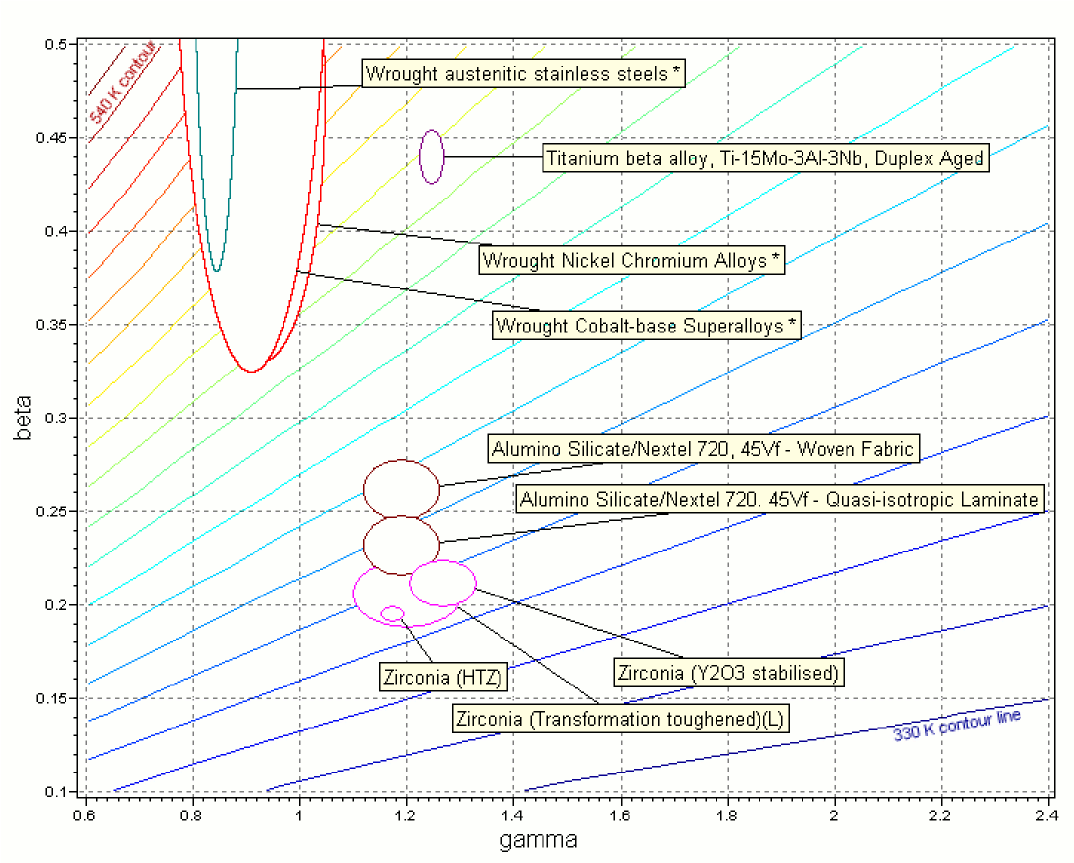

Figure 7: Thermal comparison of materials suitable for the Web using the contour plot of the maximum BFS temperature RSA. The ITPS dimensions are fixed to the values in Table 1 and BFS material is fixed to Aluminum alloy 2024. An * denotes generic material names regrouping several actual materials.

8. Conclusions

The present paper gave an illustration of how dimensional analysis can be applied to significantly reduce the number of variables used for a response surface approximation (RSA). Finite element analyses model a set of underlying equations which can be nondimensionalized, thus reducing the number of parameters describing the problem. This can greatly help in constructing a response surface approximation function of fewer variables.

Often though nondimensionalzing the exact equations involved in the finite element analysis can be relatively cumbersome or lead to too many nondimensional parameters. In this case the process can be aided by simplifying assumptions or a global sensitivity analysis that help further reduce the number of nondimensional parameters by keeping only the parameters that control most of the variation of the quantity of interest in the original problem. It is important to note that the simplifying assumptions are made only to determine the nondimensional parameters but the FE model used to construct the experiments for the RSA does not incorporate them, thus providing additional robustness.

In the presented example this approach was applied with success to a thermal heat transfer problem for an integrated thermal protection system (ITPS) and dimensional analysis in combination with several simplifying assumptions and a global sensitivity analysis allowed to reduce the number of parameters of the response surface approximation of the maximum temperature from 15 to only 2. The RSA was tested with the 15 variables being allowed to vary within relatively large bounds and the error entailed by doing so was less than 14.7 degrees K (for a range of the RSA of 250K) when compared to the accurate FE model not involving any of the simplifying assumptions. The two-dimensional RSA was used for material comparison and selection for the ITPS panel among hundreds of possible materials and for the range of variables and materials considered on the application example the maximum error in the RSA was 7.6K.

Acknowledgments

The authors would like to thank Tushar Goel for his assistance with global sensitivity analysis. They also gratefully acknowledge financial support by the NASA CUIP (formerly URETI) Grant NCC3-994 to the Institute for Future Space Transport (IFST) at the University of Florida. The cognizant program manager is Claudia Meyer at NASA Glenn Research Center.

References

[1] Vashy, A., "Sur les lois de similitude en physique", Annales télégraphiques, vol. 19, pp. 25, 1892

[2] Buckingham, E., "On Physically Similar Systems; Illustration of the Use of Dimensional Equations", Phys. Review 4, pp. 345-376, 1914

[3] Kaufman, M., Balabanov, V., Grossman, B., Mason, W. H., Watson, L. T., and Haftka, R. T., "Multidisciplinary Optimization via Response Surface Techniques," Proceedings of the 36th Israel Conference on Aerospace Sciences (Israel), Omanuth, Haifa, Israel, 1996, pp. A-57–A-67

[4] Vignaux, G.A., Scott, J.L., "Simplifying regression models using dimensional analysis", Austr. & New Zealand J. Stat., Vol. 41(1), pp. 31-41,1999

[5] Lacey, D. and Steele, C., "The use of dimensional analysis to augment design of experiments for optimization and robustification", Journal of Engineering Design Vol.17(1), pp. 55-73, 2006

[6] Venter, G., Haftka, R.T. and Starnes, J.H, "Construction of response surface approximations for design optimization", Aiaa Journal, Vol. 36(12), pp. 2242-2249, 1998

[7] Bapanapalli, S.K., Martinez, O.M., Gogu, C., Sankar, B.V., Haftka, R.T., Blosser, M.L., "Analysis and design of corrugated core sandwich panels for thermal protection systems of space vehicles", AIAA Paper 2006-1942, *47th AIAA/ASME/ ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Newport, May 2006

[8] Myers, D., Martin, C., Blosser, M.L., "Parametric Weight Comparison of Advanced Metallic, Ceramic Tile, and Ceramic Blanket Thermal Protection Systems (TPS)", NASA technical report, 1998

[9] Blosser, M.L., "Development of Advanced Metallic, Thermal-Protection-System Prototype Hardware", *Journal of Spacecraft and Rockets*, Vol. 41, No. 2, Mar-Apr 2004, pp 183-194

[10] Poteet, C.C., Abu-Khajeel, H., Hsu, S-Y, "Preliminary thermal-mechanical sizing of a metallic thermal protection system," *Journal of Spacecraft and Rockets*, Vol. 41, No. 2, Mar – Apr 2004, pp. 173-182

[11] Sobol, I. M., "Sensitivity Estimates for Nonlinear Mathematical Models," *Mathematical Modeling and Computational Experiment*, New York, NY, John Wiley & Sons, Vol. 1, 1993, pp 407-414

[12] Elishakoff, I., "Convex Versus Probabilistic Modeling of Uncertainty in Structural Dynamics," *Structural Dynamics Recent Advances*, edited by M. Petyt, H. F. Wolfe, and C. Mei, Keynote Lecture, Elsevier, London,1991, pp. 3–21

[13] Elishakoff, I., Haftka, R.T., Fang, J., "Structural design under bounded uncertainty-optimization with anti-optimization", *Computers & structures*, vol. 53, no 6, 1994, pp. 1401-1405

[14] Gogu, C., Bapanapalli, S.K., Haftka, R.T., Sankar, B.V., "Comparison of Materials for Integrated Thermal Protection Systems for Spacecraft Reentry", AIAA Paper 2007-1860, *3rd AIAA Multidisciplinary Design Optimization Specialist Conference*, Honolulu, April 2007